Continuous Aperture Phased MIMO:
Basic Theory and Applications

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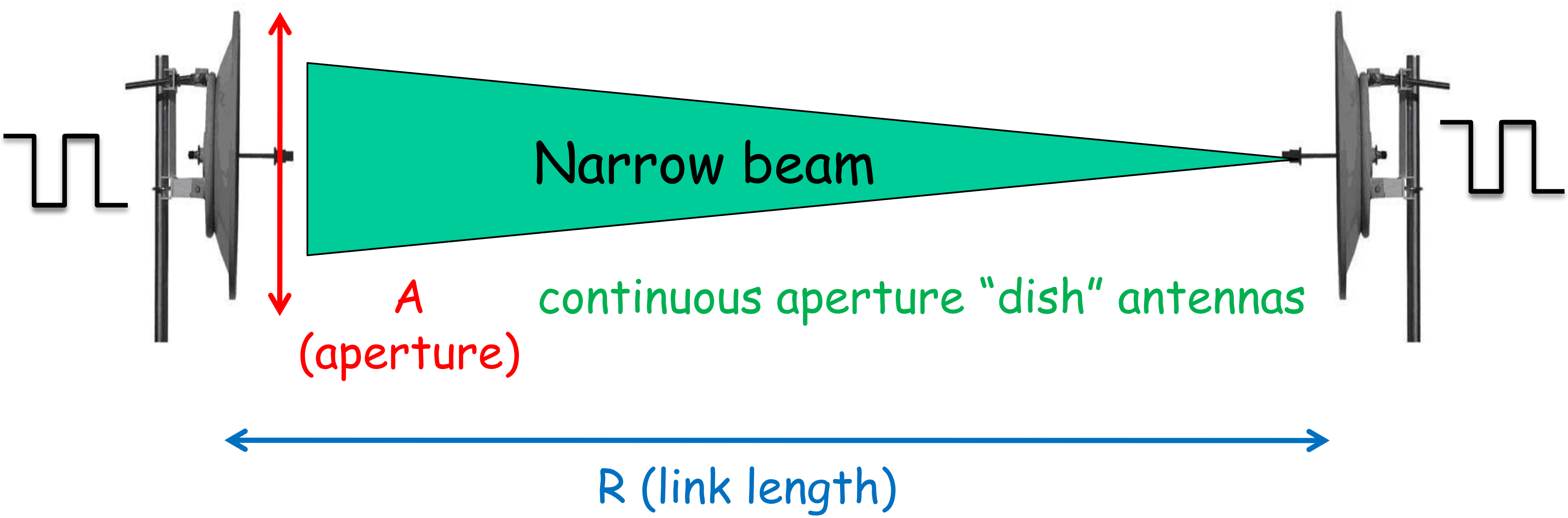
mm-wave MIMO

- Wireless bandwidth requirements are exploding (I-phones, I-pads, video)
- mm-wave systems provide a unique synergistic opportunity
  - 60-100 GHz; large bandwidths (GHz)
  - MIMO operation with compact arrays; short wavelengths (3-5mm)
- mm-wave line-of-sight (LoS) links (Gigabits/s speeds)
  - Wireless backhaul; alternative to fiber for connecting wireless traffic to backbone internet
  - Indoor wireless links (e.g., HDTV)
  - Smart basestations
- State-of-the-art:
  - Traditional DISH systems with continuous-aperture "dish" antennas
  - MIMO systems that use discrete arrays

DISH System



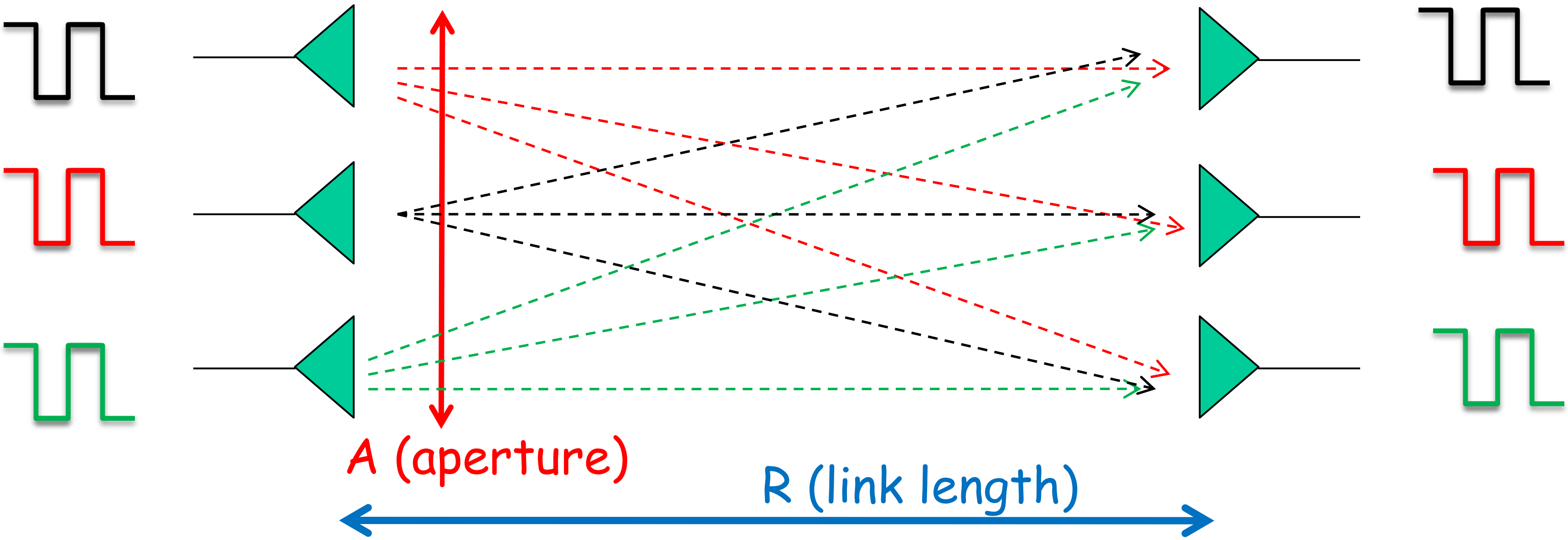
Pros: Large array gain (SNR gain) proportional to $n \approx 2A/\lambda_c$

Narrow beam (continuous aperture)

Cons: Single data stream

MIMO System

Discrete Antenna Arrays



Pros: Multiple data streams (spatial multiplexing)

(# data streams limited by A and R)

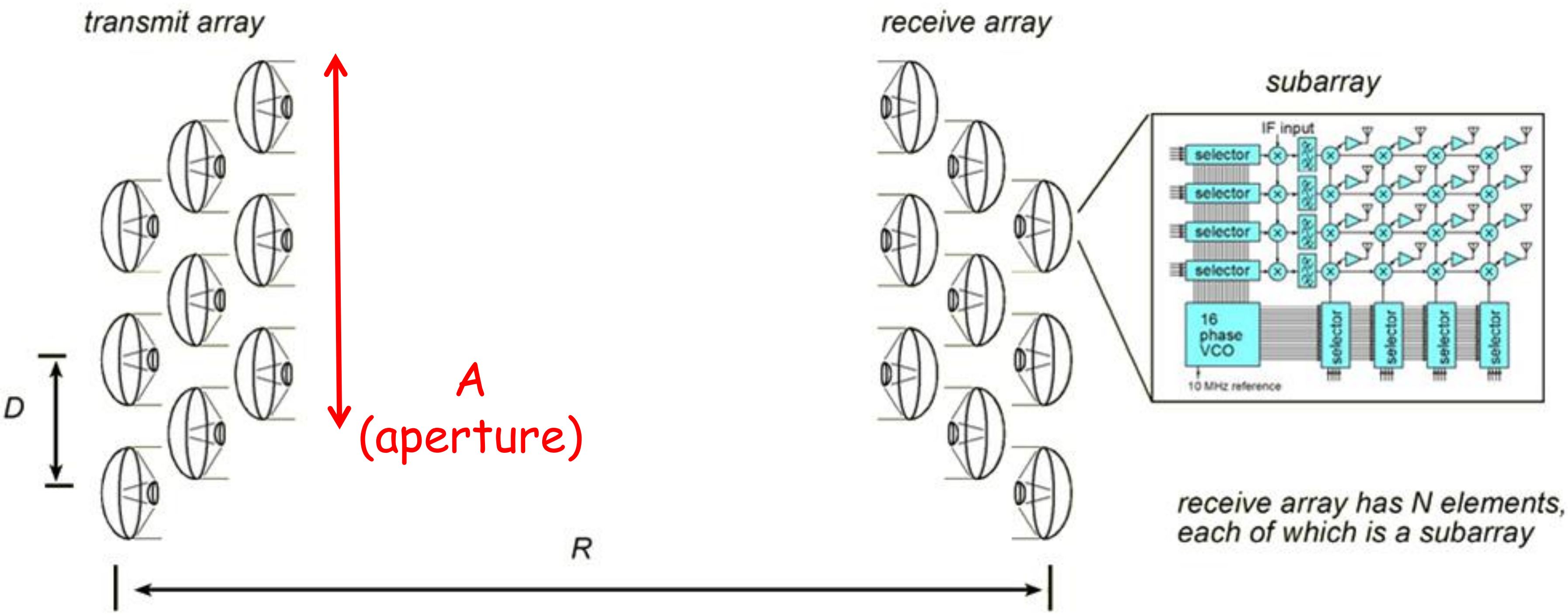
Cons: Reduced array gain (due to discrete antenna elements)

Grating lobes

MIMO System

(Madhow, Rodwell et. al; Allerton 2006)

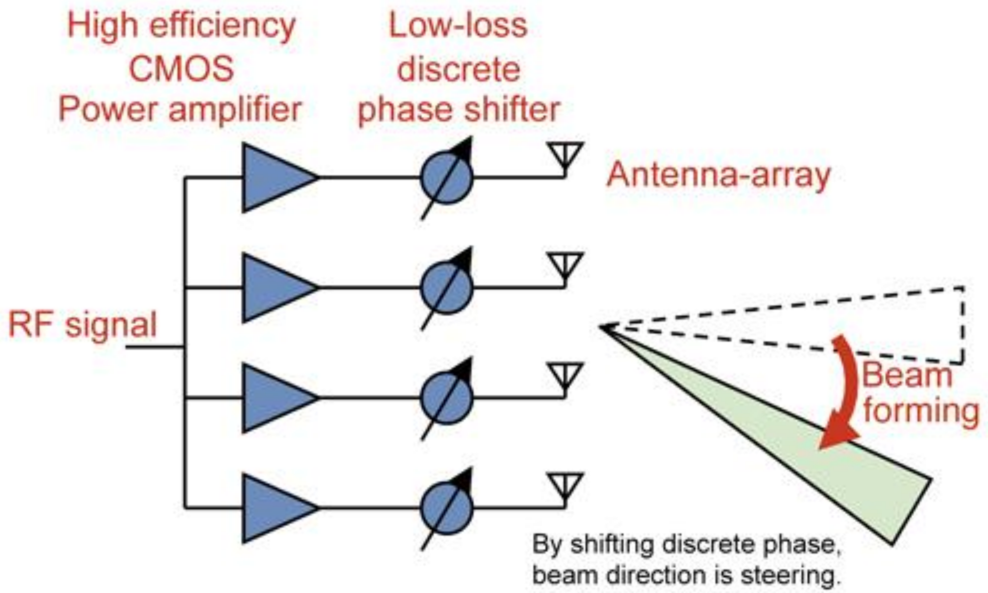
(UCSB architecture)



Bohagen, Orten, Oien 07'

Chalmers

Phased Arrays (Beam Steering)

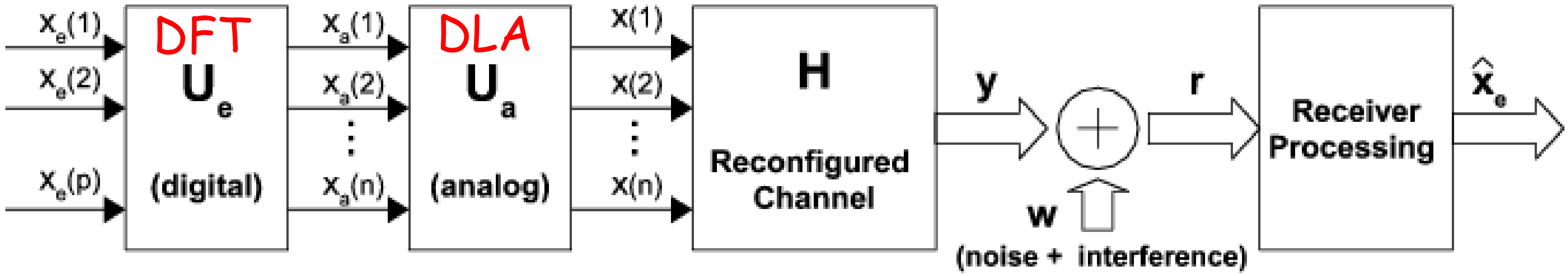


(Tohoku University)

MIMO can also be implemented as a discrete phased-array

Implementation is challenging for large number of antenna elements

CAP-MIMO: Hybrid Analog-Digital Transceiver



Analog component: High-resolution Discrete Lens Array (DLA)
Analog beamforming

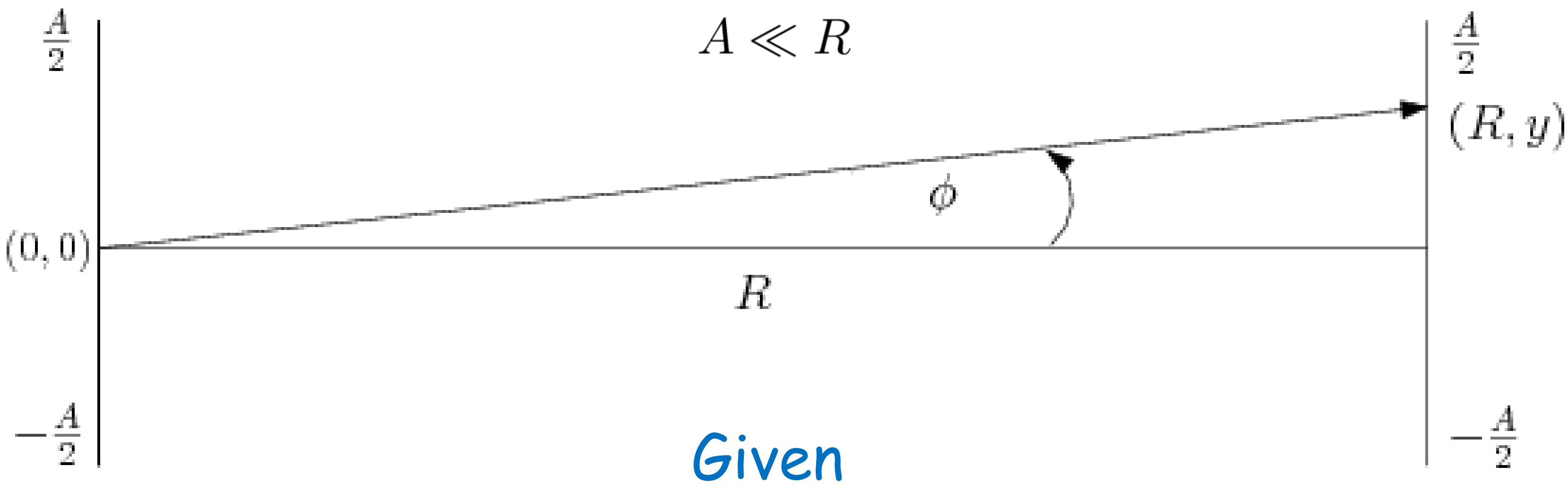
Digital processor: Oversampled discrete Fourier Transform (DFT)
stable interface to the analog front-end

Digital modes: p = number of (spatial) data streams

Analog modes: n >> p (continuous aperture)

Compelling performance gains over state-of-the-art:
- Capacity and power efficiency
- D/A complexity

Motivating Questions



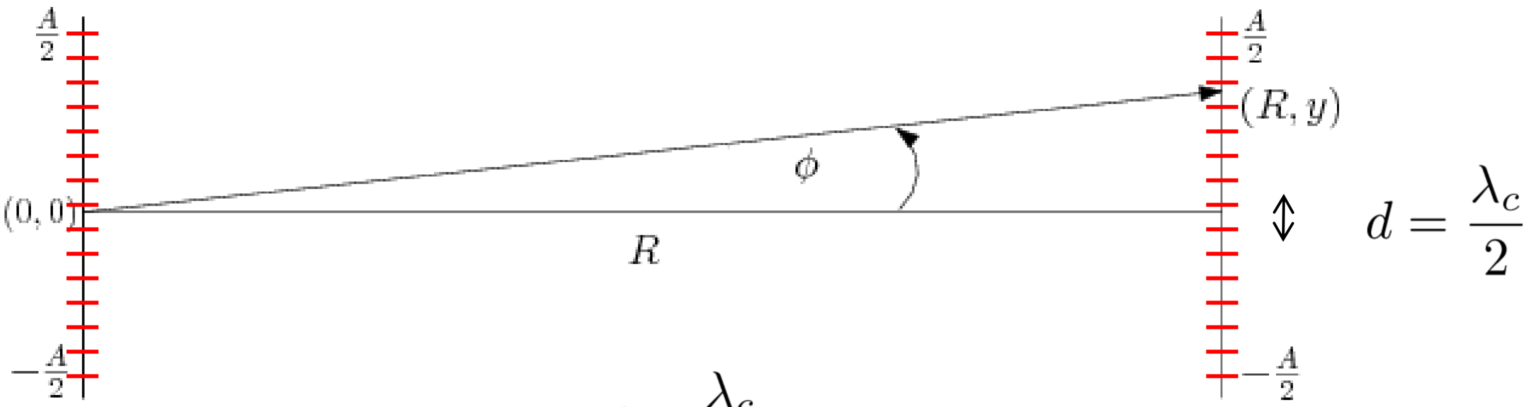
Given

$$(R, A, \lambda_c)$$

What's the capacity of a LoS link at any operating SNR?

How do we approach link capacity in practice?

Critically Sampled LoS Channel



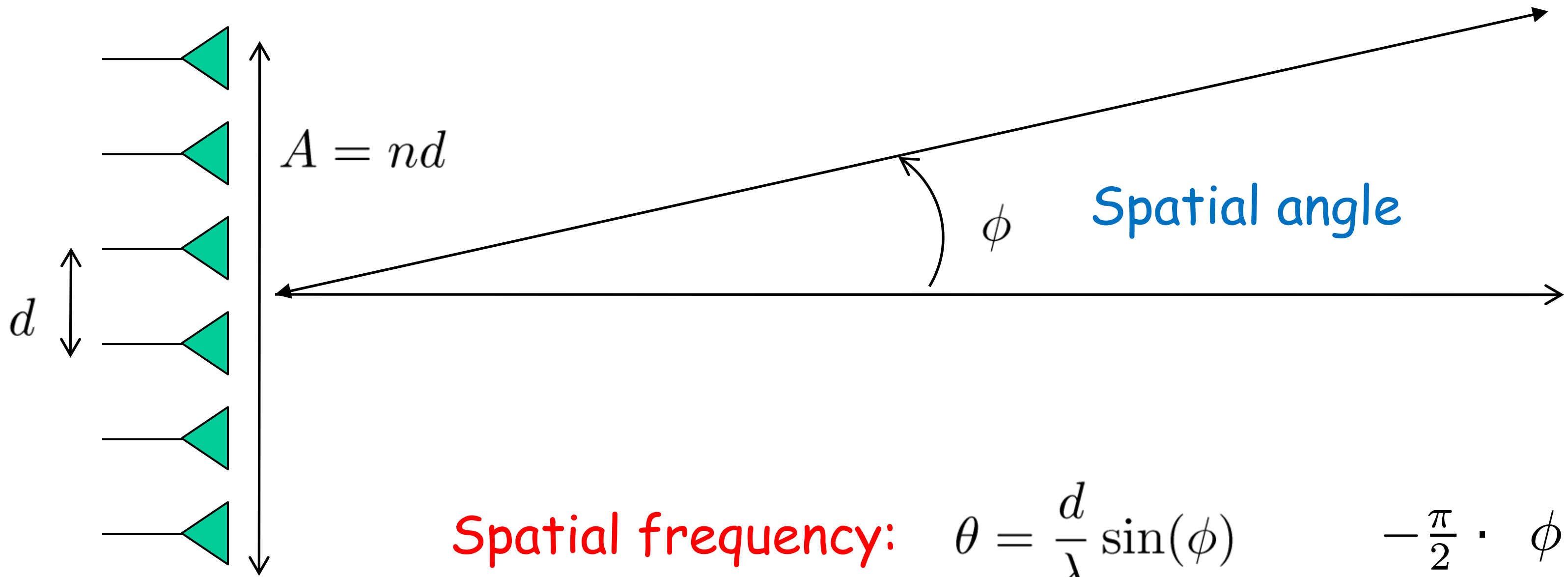
$$d = \frac{\lambda_c}{2}$$

analog spatial modes: $n \approx \frac{A}{d} = \frac{2A}{\lambda_c}$

n x n MIMO system: $\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{w}$

Beamspace characterization of H: coupled phased arrays

n-element Phased Array



Spatial frequency: $\theta = \frac{d}{\lambda}\sin(\phi)$ $\quad -\frac{\pi}{2} \cdot \phi \cdot \frac{\pi}{2}$

Array steering (tx) or response (rx) vector:

(n-dimensional spatial sinusoid)

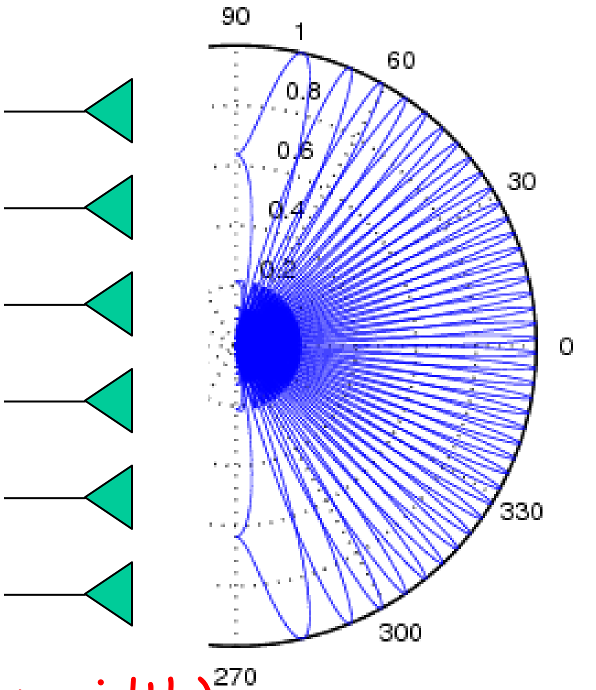
$$\mathbf{a}_n(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi\theta} \\ \vdots \\ e^{-j2\pi\theta(n-1)} \end{bmatrix} \qquad -\frac{1}{2} \cdot \theta \cdot \frac{1}{2}$$

Critically-spaced Phased Array (modes = beams)

n = 40

$$d = \frac{\lambda_c}{2} \Longrightarrow \theta = 0.5 \sin(\phi)$$

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2} \Longleftrightarrow -\frac{1}{2} \leq \theta \leq \frac{1}{2}$$



n orthogonal spatial beams/modes

$$\theta_i = i\Delta\theta_o = \frac{i}{n} \qquad i = 0, \cdots, n-1$$

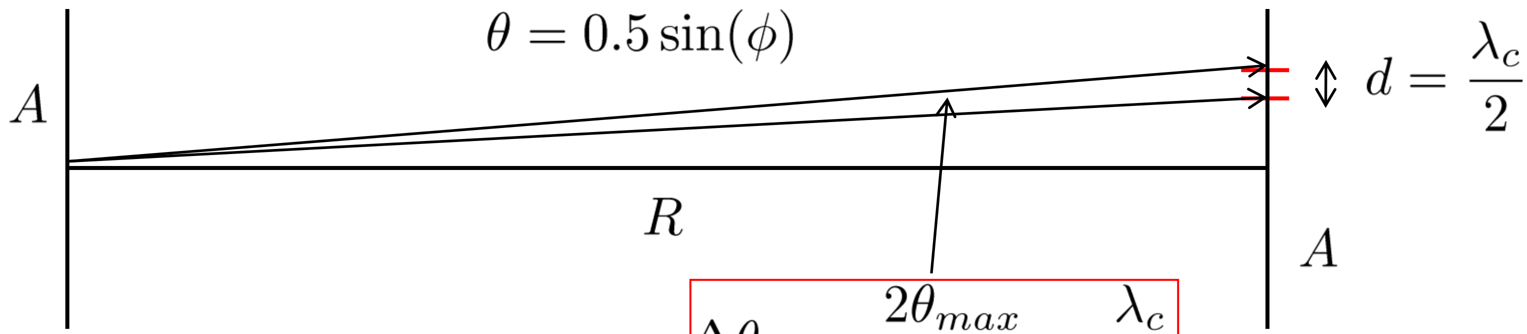
$$\Delta\theta_o = \frac{1}{n} \longleftrightarrow \Delta\phi_o = \frac{\lambda_c}{A}$$

(spatial resolution/beamwidth)

DFT matrix:
n-dimensional
orthogonal basis

$$\mathbf{U}_n = \frac{1}{\sqrt{n}} \left[\mathbf{a}_n(\theta_o), \mathbf{a}_n(\theta_1), \cdots, \mathbf{a}_n(\theta_{n-1})\right]$$

Channel Induced by Critically Sampled Apertures



$$\Delta\theta_{ch} = \frac{2\theta_{max}}{n} = \frac{\lambda_c}{4R}$$

$$\mathbf{a}_n(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi\theta} \\ \vdots \\ e^{-j2\pi\theta(n-1)} \end{bmatrix}$$

$$\theta_i = i\Delta\theta_{ch} = i\frac{\lambda_c}{4R} \quad i = 0, \cdots, n-1$$

n x n (all-phase) channel matrix: $\mathbf{H} = [\mathbf{a}_n(\theta_o), \cdots, \mathbf{a}_n(\theta_{n-1})]$

Channel power: $\sigma_c^2 = \mathrm{tr}\left(\mathbf{H}^H\mathbf{H}\right) = n^2$

$$\Delta\theta_{ch} = \sin(\phi_{max})\Delta\theta_o \approx \frac{A}{2R}\Delta\theta_o \ll \Delta\theta_o$$

Digital Modes (Multiplexing Gain): Coupled Orthgonal Beams

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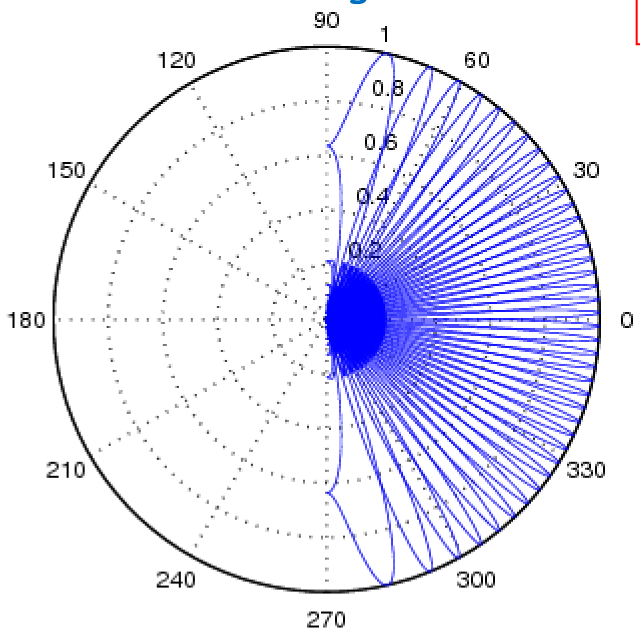
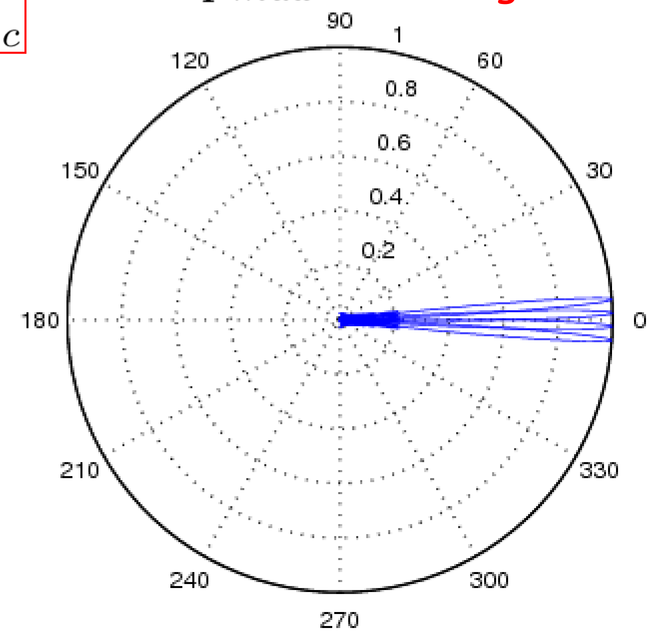
$$p_{max} = \frac{A^2}{R\lambda_c}$$

n = 40 analog modes



Finite Receiver aperture

$p_{max} = 4$ digital modes



$$2\theta_{max} = \sin(\phi_{max}) \approx \frac{A}{2R}$$

Maximum number of digital modes:

$$p_{max} = \frac{2\theta_{max}}{\Delta\theta_o} = 2\theta_{max} n = \frac{A^2}{R\lambda_c}$$

Link Capacity: Exact Approach

n x n MIMO system: $\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{w}$ $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$

H is deterministic – known at TX and RX

Capacity-achieving input is Gaussian

$$\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{V}\boldsymbol{\Lambda}_s\mathbf{V}^H\right)$$

Total TX SNR: $\mathrm{tr}(\boldsymbol{\Lambda}_s) = \sum_{i=1}^{n} \rho_i = \rho$

Transmission on transmit eigenvectors

$$\boldsymbol{\Sigma}_{tx} = \mathbf{H}^H\mathbf{H} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^H$$

$$\mathrm{tr}(\boldsymbol{\Lambda}) = \sum_{i=1}^{n} \lambda_i = \sigma_c^2 = n^2$$

Link Capacity: Exact Approach

n x n MIMO system: $\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{w}$ $\quad \mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$

H is deterministic - known at TX and RX

$$C(\rho) = \max_{\boldsymbol{\Lambda}_s : \mathrm{tr}(\boldsymbol{\Lambda}_s) = \rho} \log |\mathbf{I} + \boldsymbol{\Lambda}\boldsymbol{\Lambda}_s| = \max_{\rho_i : \sum_i \rho_i = \rho} \sum_{i=1}^{n} \log(1 + \lambda_i \rho_i)$$

$(p_{eff} \sim p_{max}$ digital modes$)$

$$\approx \max_{\rho_i : \sum_i^{p_{eff}} \rho_i = \rho} \sum_{i=1}^{p_{eff}} \log(1 + \lambda_i \rho_i)$$

(equal transmit power allocation)
$(\rho_i = \rho / p_{eff})$

$$\geq \sum_{i=1}^{p_{eff}} \log\left(1 + \lambda_i \frac{\rho}{p_{eff}}\right)$$

(equal channel power allocation)
$(\lambda_i = \sigma_c^2 / p_{eff} = n^2 / p_{eff})$

$$\approx p_{eff} \log\left(1 + \rho \frac{n^2}{p_{eff}^2}\right)$$

CAP-MIMO Capacity: Accurate Closed-form Approximations

$$C(\rho) \approx p_{max} \log(1 + \rho_{rx}) = p_{max} \log\left(1 + \rho \frac{n^2}{p_{max}^2}\right)$$

$$p_{max} = \text{ number of digital modes (multiplexing gain)}$$

$$\rho_{rx} = \text{ RX SNR per mode} = \rho \frac{n^2}{p_{max}^2}$$

(large SNR gain)

MIMO interpretation:

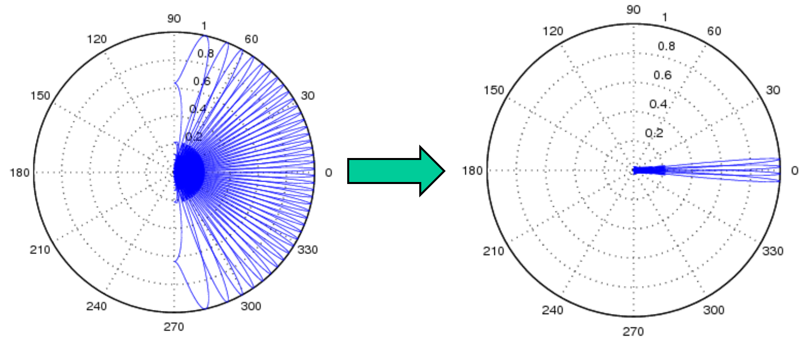
$$\rho_{rx} = \frac{\rho}{p_{max}} \times \frac{\sigma_c^2}{p_{max}}$$

TX SNR/mode x channel power/mode

Phased-array interpretation:

$$\rho_{rx} = \frac{\rho}{p_{max}} \times n \times \frac{n}{p_{max}}$$

TX SNR/mode x TX array gain x RX array gain



n Analog modes

p_{max} Digital modes

Transmit-Receive Array Gain

Orthogonal beams: $\theta_i = i\Delta\theta_o = \frac{i}{n}$

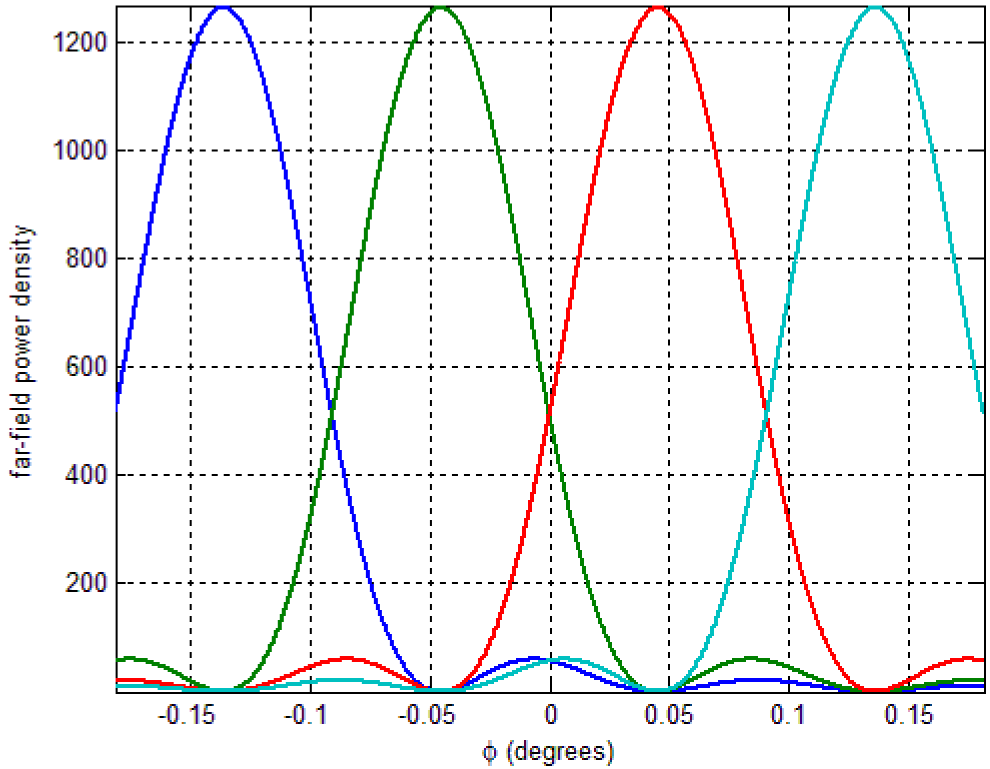
$i = 0, \cdots, n-1$

Far-field power density: $|b_i(\theta)|^2$

$$b_i(\theta) = \frac{1}{\sqrt{n}} \mathbf{a}_n^H(\theta)\mathbf{a}_n(\theta_i)$$

$$= \frac{1}{\sqrt{n}} \frac{\sin(\pi n(\theta - \theta_i))}{\sin(\pi(\theta - \theta_i))}$$

TX array gain: n-fold (compared to omni-directional antennas)



$n = 1245;\ p_{max} = 4;$

Narrow beams: $\Delta\theta_o = \frac{1}{n} \longleftrightarrow \Delta\phi_o = \frac{\lambda_c}{A} \quad \Rightarrow R\Delta\phi_o = \frac{R\lambda_c}{A} = \frac{A}{p_{max}} \Leftrightarrow \frac{n}{p_{max}}$ (Rx gain)

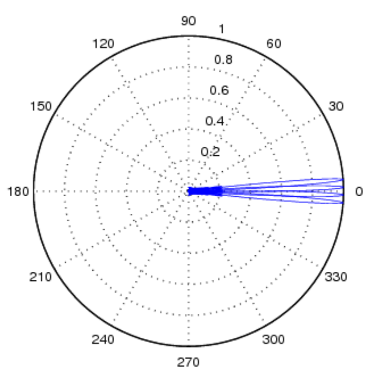
DISH System Capacity

(no multiplexing gain, large SNR gain)

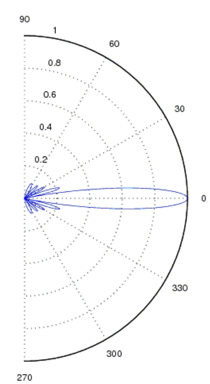
$$\log\left(1+\rho\frac{n^2}{p_{max}}\right) \leq C_{dish}(\rho) \leq \log\left(1+\rho n^2\right)$$

$p_{max} > 1$

$p_{max} = 1$



p_{max} Digital modes



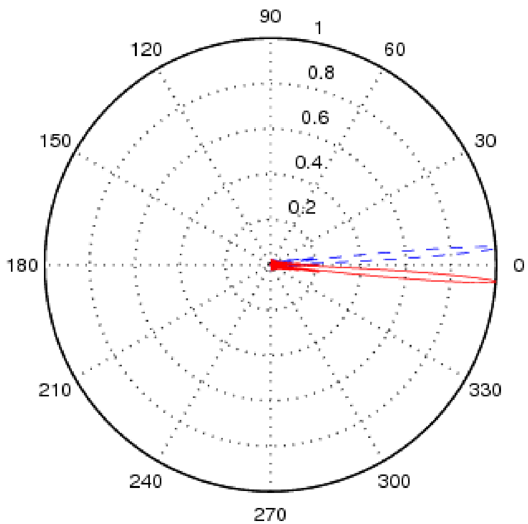
MIMO System Capacity

(maximum multiplexing gain, no/small SNR gain)

$$C_{mimo}(\rho) = p_{max} \log\left(1 + \rho \frac{\sigma_c^2}{p_{max}^2}\right) = p_{max} \log(1+\rho) \qquad \sigma_c^2 = p_{max}^2$$

Uses p_{max} antennas with (Rayleigh) spacing: $d_{ray} = \sqrt{\frac{R\lambda_c}{p_{max}}}$

Power loss: Grating lobes

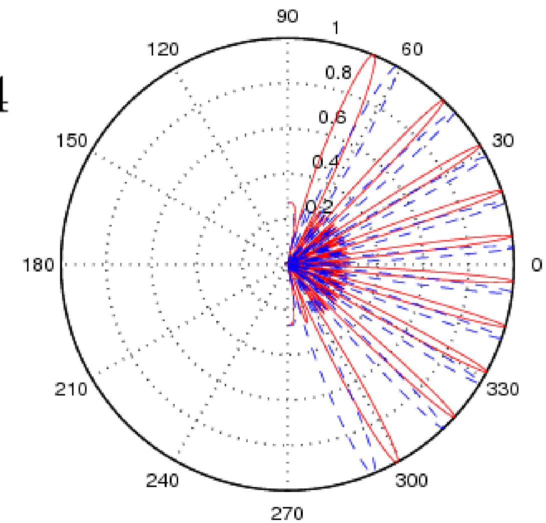


CAP-MIMO beampatterns

$$n = 40, \ p_{max} = 4$$

$$n_c = \frac{n}{p_{max}}$$

$n_c - 1$ grating lobes for each beam



MIMO beampatterns

Capacity Summary

MIMO: $$C_{mimo}(\rho) = p_{max} \log(1+\rho)$$

DISH: $$\log\left(1+\rho\frac{n^2}{p_{max}}\right) \leq C_{dish}(\rho) \leq \log\left(1+\rho n^2\right)$$

CAP-MIMO: $$C(\rho) \approx p_{max} \log\left(1+\rho\frac{n^2}{p_{max}^2}\right)$$

SNR gain over MIMO: $$G = \frac{n^2}{p_{max}^2}$$

Multiplexing gain over DISH: p_{max}

2D Square Apertures

$A \times A$ square aperture

$$\mathbf{H}_{2d} = \mathbf{H} \otimes \mathbf{H}$$

Analog modes: $n_{2d} = n^2 \ , \ n \approx \frac{2A}{\lambda_c}$

Digital modes: $p_{max,2d} = p_{max}^2 \ , \ p_{max} \approx \frac{A^2}{R\lambda_c}$

Transmit Covariance Matrix:

$$\mathbf{\Sigma}_{tx,2d} = \mathbf{H}_{2d}^H \mathbf{H}_{2d} = \mathbf{V}_{2d} \mathbf{\Lambda}_{2d} \mathbf{V}_{2d}^H$$

$$\mathbf{V}_{2d} = \mathbf{V} \otimes \mathbf{V} \qquad \mathbf{\Lambda}_{2d} = \mathbf{\Lambda} \otimes \mathbf{\Lambda}$$

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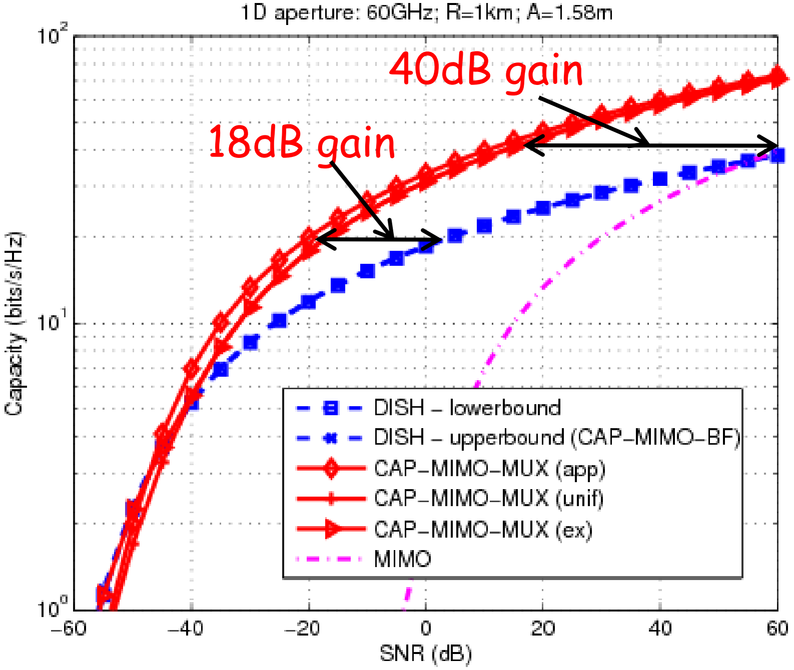
Potential Performance Gains

$$f_c = 60\text{GHz}\ ;\ \lambda_c = 5\text{mm}$$

1D (linear) aperture

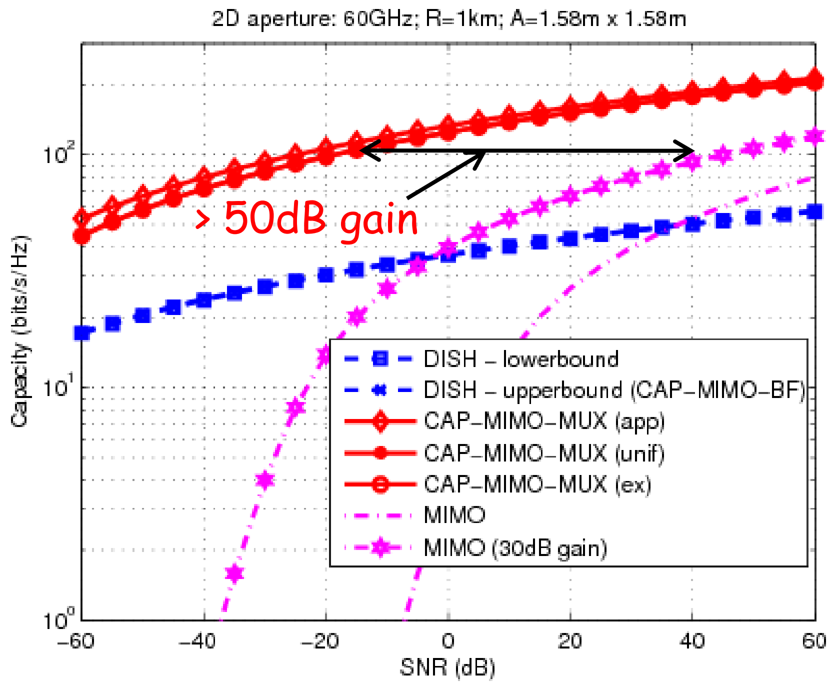


$$R = 1\text{km}\ ;\ A = d_{ray} = 1.58\text{m}$$

$$n = 632\ ;\ p_{max} = 2$$ (G = 50dB)

Rate (1 GHz BW): 40 Gb/s @ SNR = 20dB

2D (square) aperture



$$R = 1\text{km}\ ;\ A = 1.58 \times 1.58\text{m}^2$$

$$n_{2d} = 399424\ ;\ p_{max,2d} = 4$$ (G = 100dB)

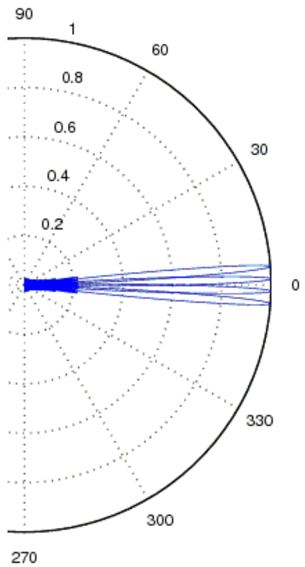
Rate (1 GHz BW): 200 Gb/s @SNR = 40dB

CAP-MIMO Configurations: Beam-Agility

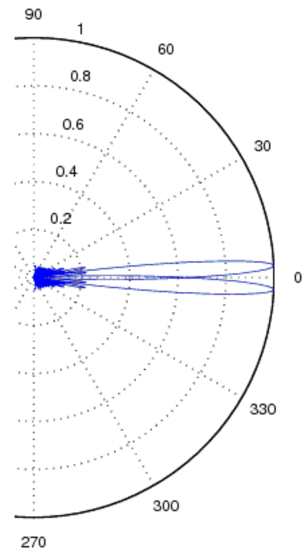
$p = 1, \cdots, p_{max}$

$$C(\rho) \approx p \log \left(1 + \rho \frac{n^2}{p p_{max}} \right)$$

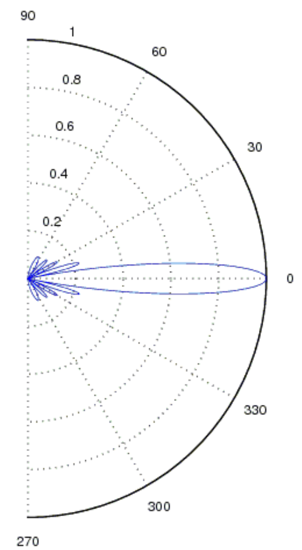
$n = 40, \; p_{max} = 4$



p=4
MUX
Low robustness
(maximum capacity)



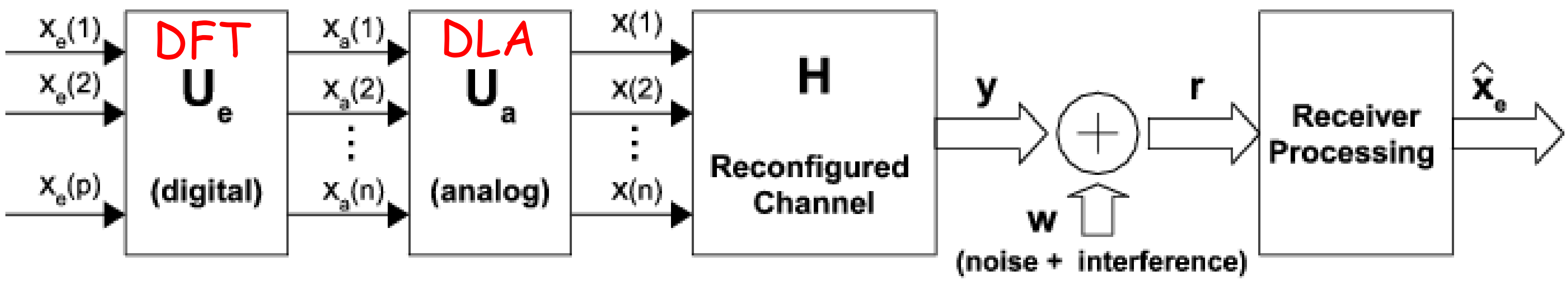
p=2
INT
Medium Robustness
(medium capacity)

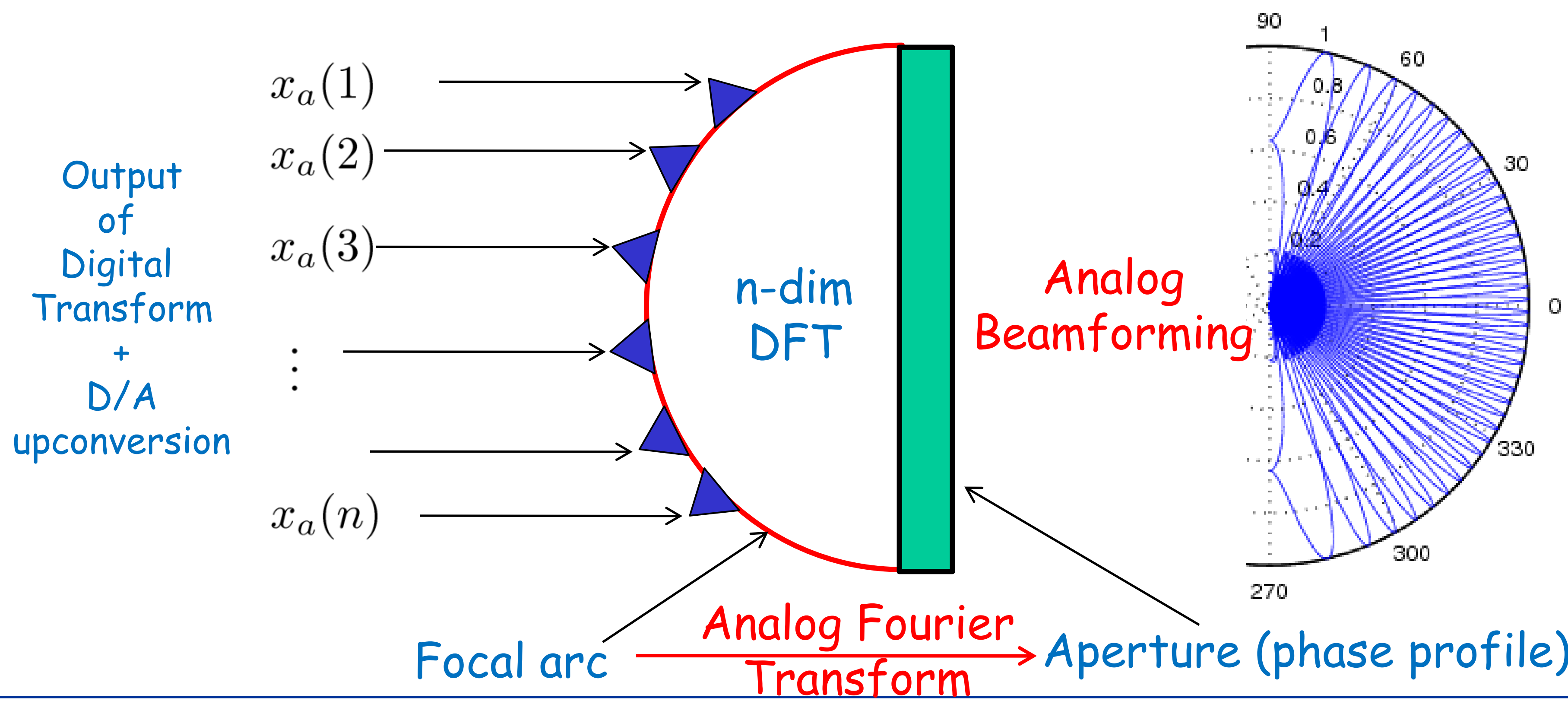


p=1
BF
High robustness
(lowest capacity)

Analog Front-End: Discrete Lens Array

(long history: e.g., McGrath; Cox; Popovic; Behdad)

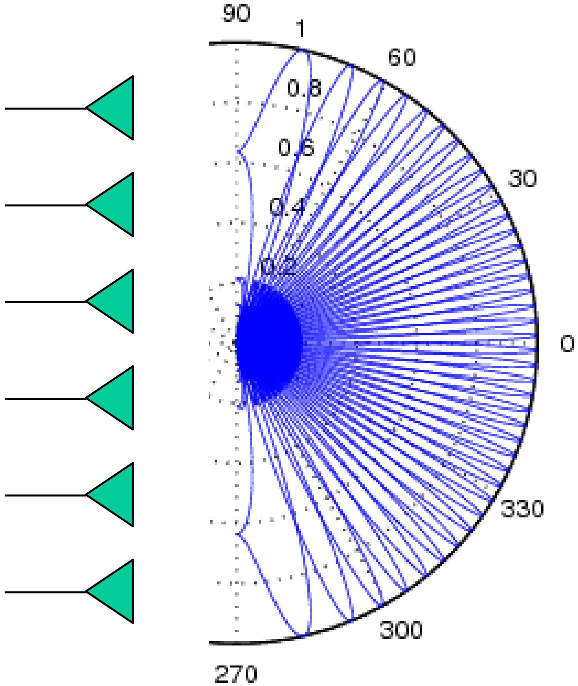




D/A Complexity

Phased Array: digital beamforming

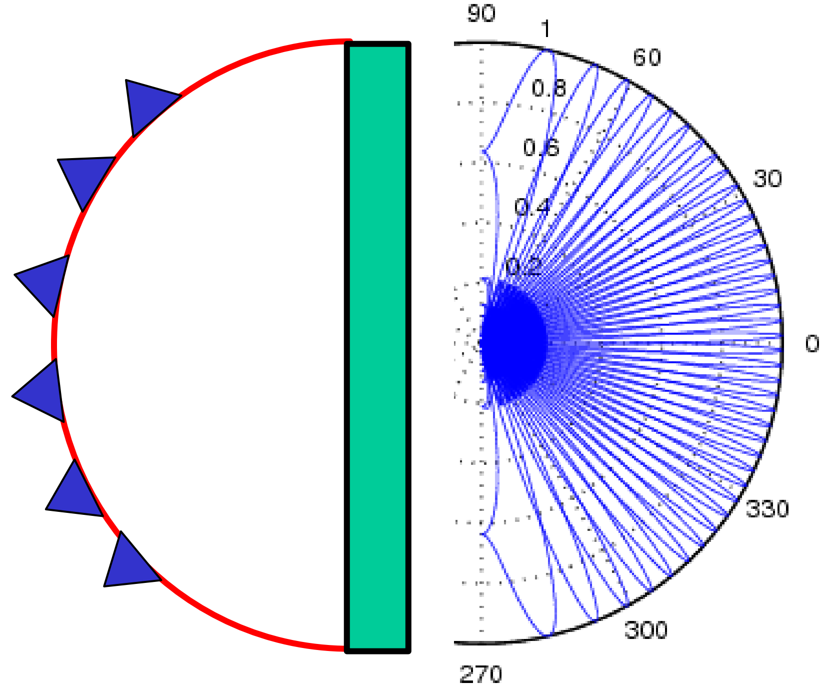
$n -$ dim. D/A interface



n active inputs

DLA: analog beamforming

$n -$ dim. D/A interface



n active inputs

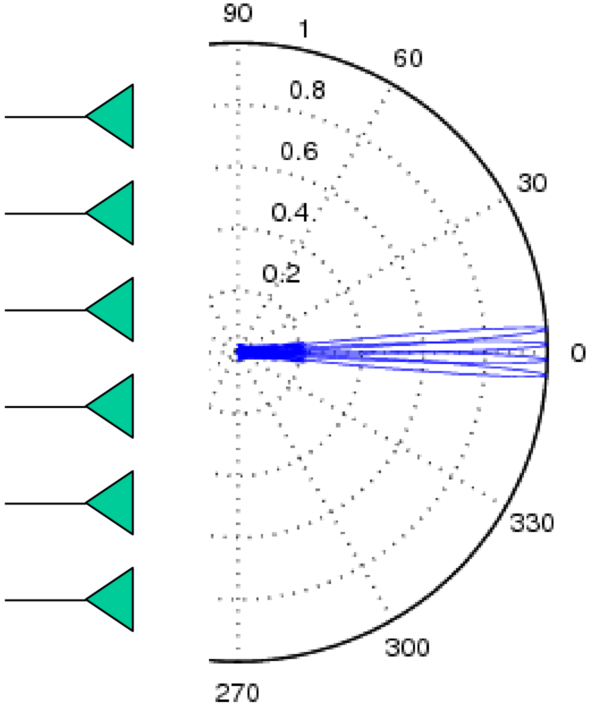
D/A Complexity

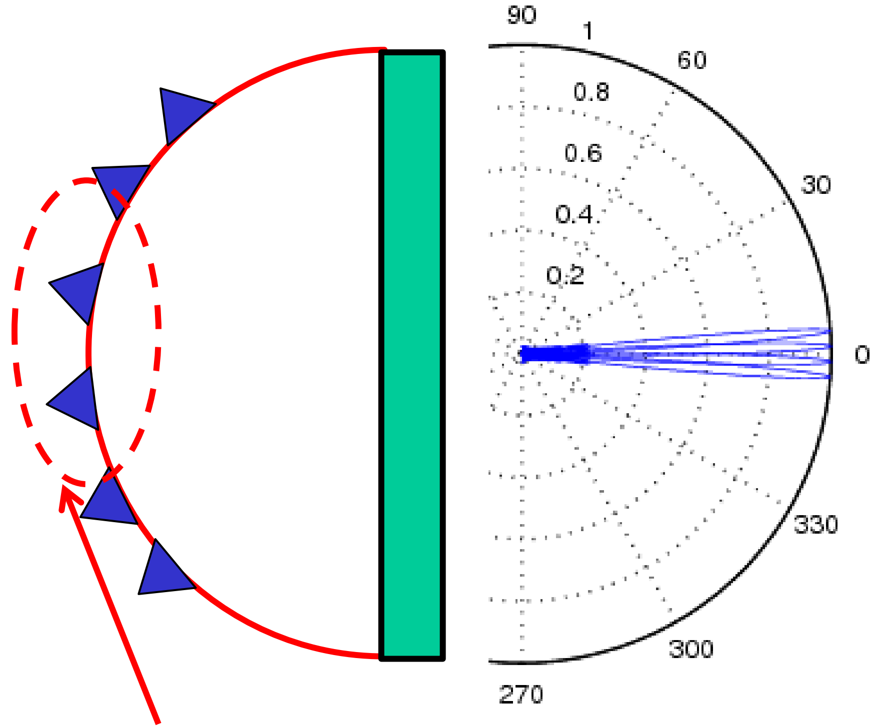
Phased Array: digital beamforming

DLA: analog beamforming

n – dim. D/A interface

$O(p_{max})$ – dim. D/A interface

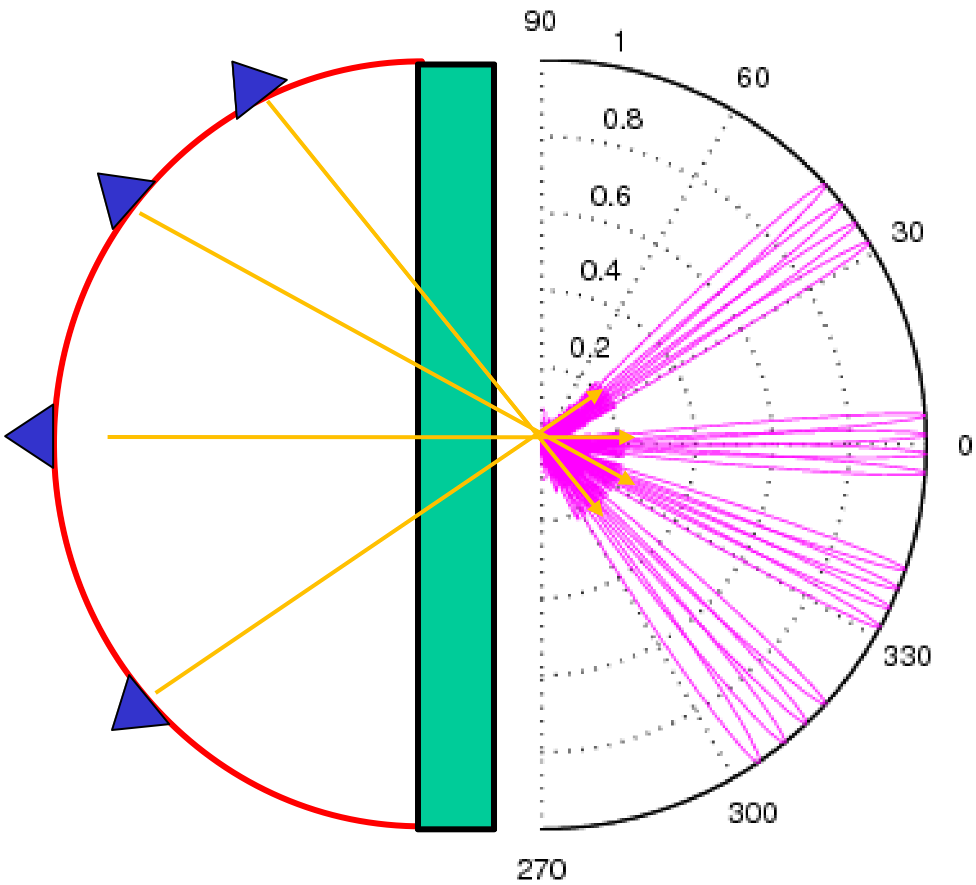




n active inputs

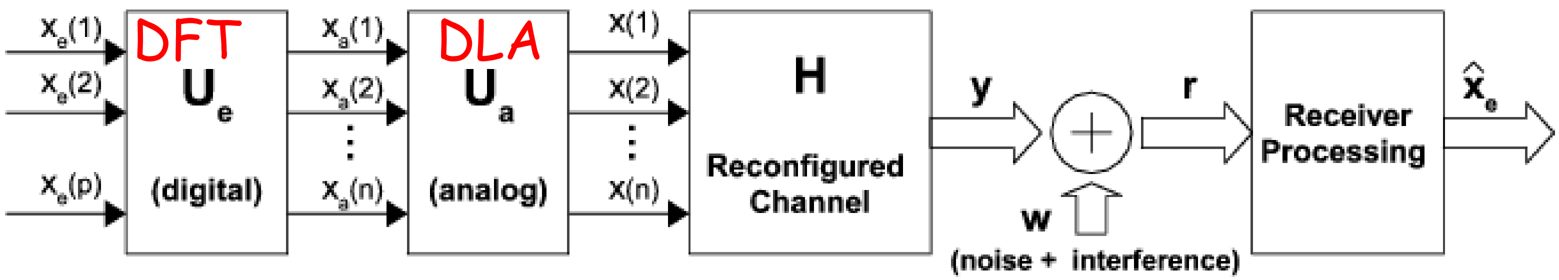
$O(p_{max})$ active inputs

Point-to-multipoint Operation



Smart basestations

Conclusion



- Hybrid Analog-Digital CAP-MIMO Transceiver
  - MIMO multiplexing gain (w/o grating lobes)
  - Power/beamforming gain of continuous apertures (DISH)
  - Beam-steering advantages of phased arrays
  - Analog beamforming: dramatically reduced-complexity A-D interface

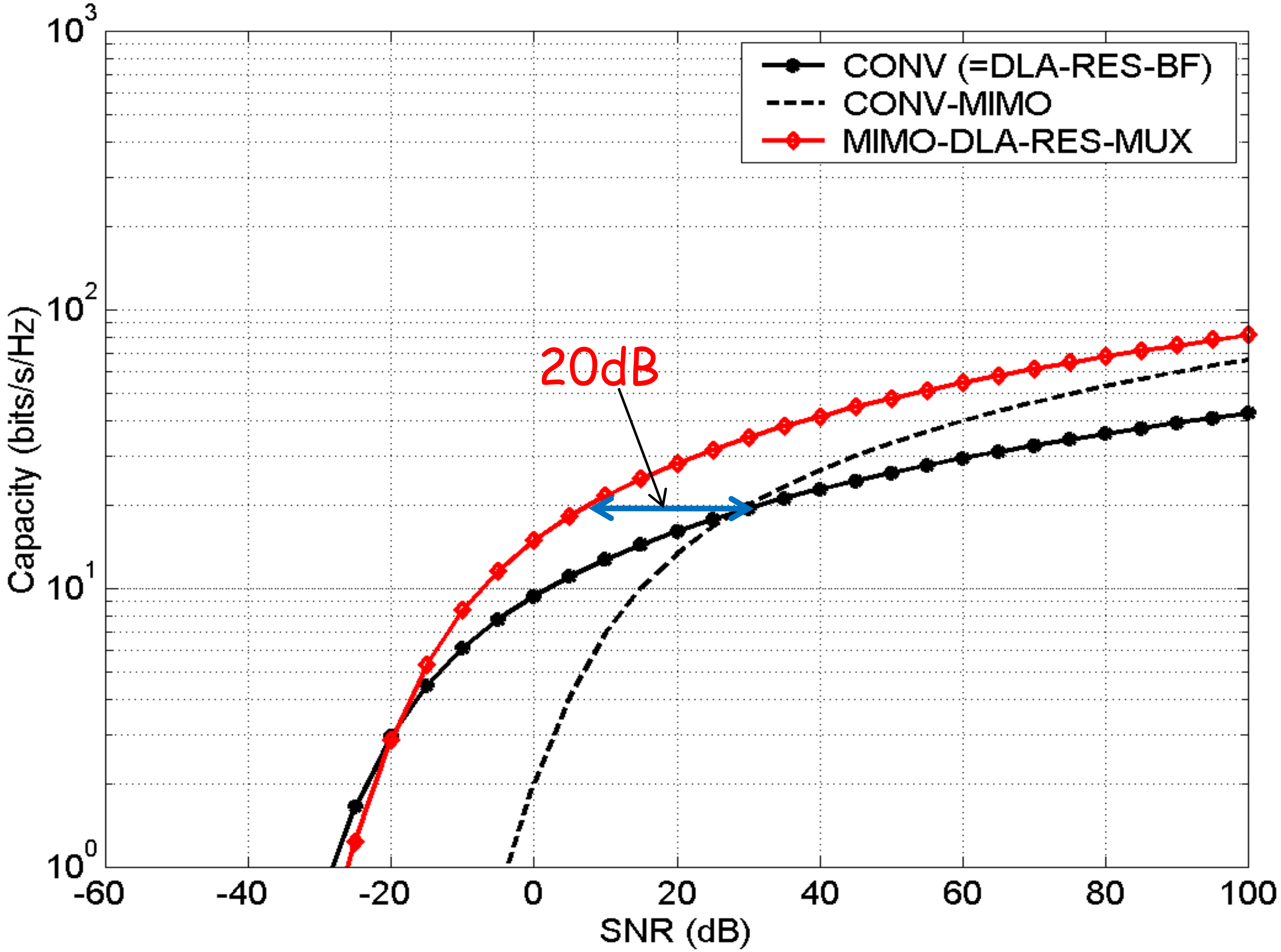
- Very compelling capacity/SNR gains over the state-of-the-art

- Timely applications
  - Long-range wireless backhaul links (> 100 Gb/s)
  - High-rate short-range links (> 10 Gb/s)
  - Smart basestations

Prototype Specifications

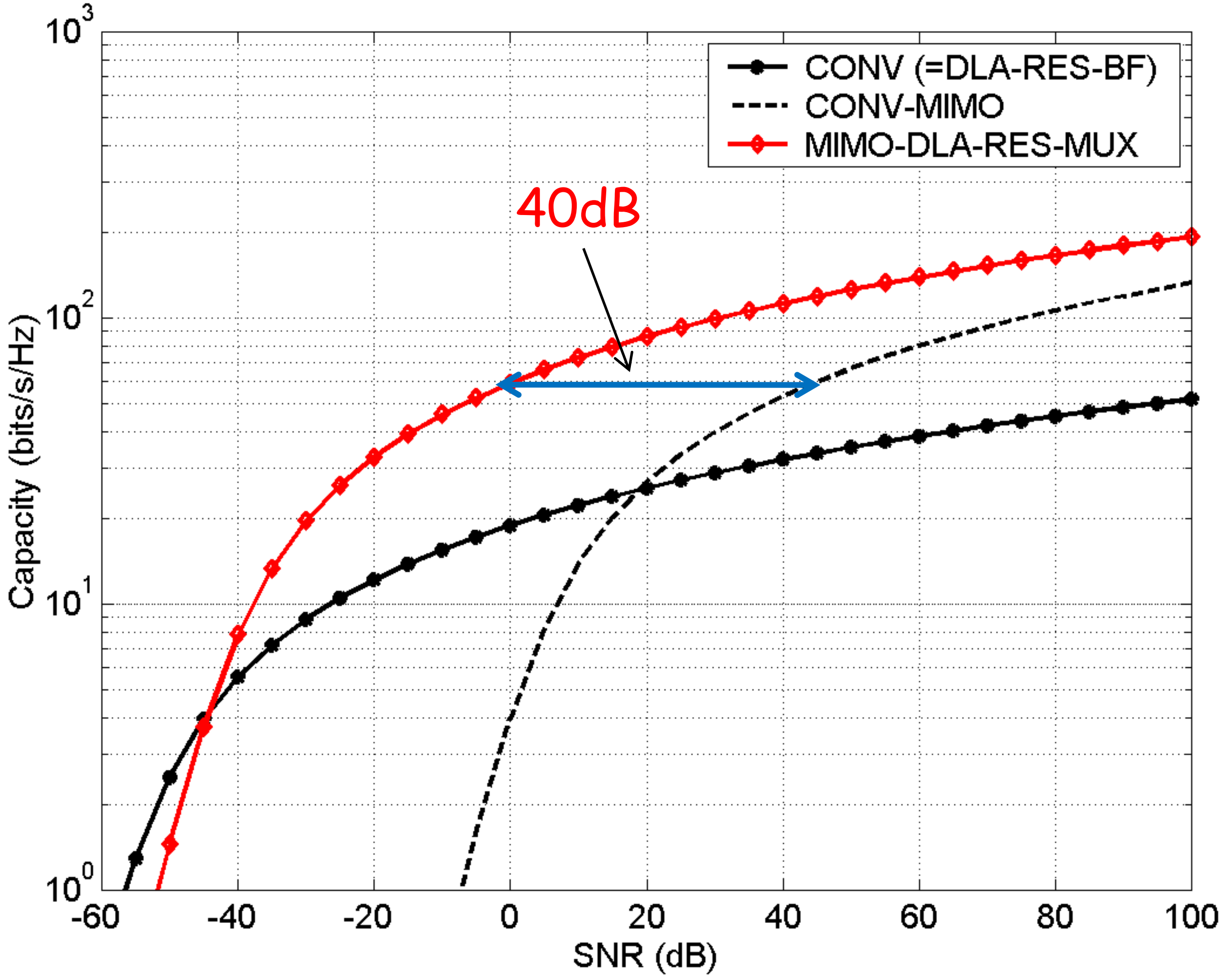
- 10 GHz carrier frequency
- Two 40cm x 40cm (aperture) DLAs, one for the transmitter and one for the receiver
- 10-20 feed antennas for exciting the focal arc of the DLAs
- 10 ft link length – with line-of-sight (LoS) propagation
- p=4 (2 x 2) , n = 676 (26 x 26)

Prototype Capacity Assessment: 1D



Linear array: p=2, n=26

Prototype Capacity Assessment: 2D



Square array: p=4, n=676 (26 x 26)

Next Steps

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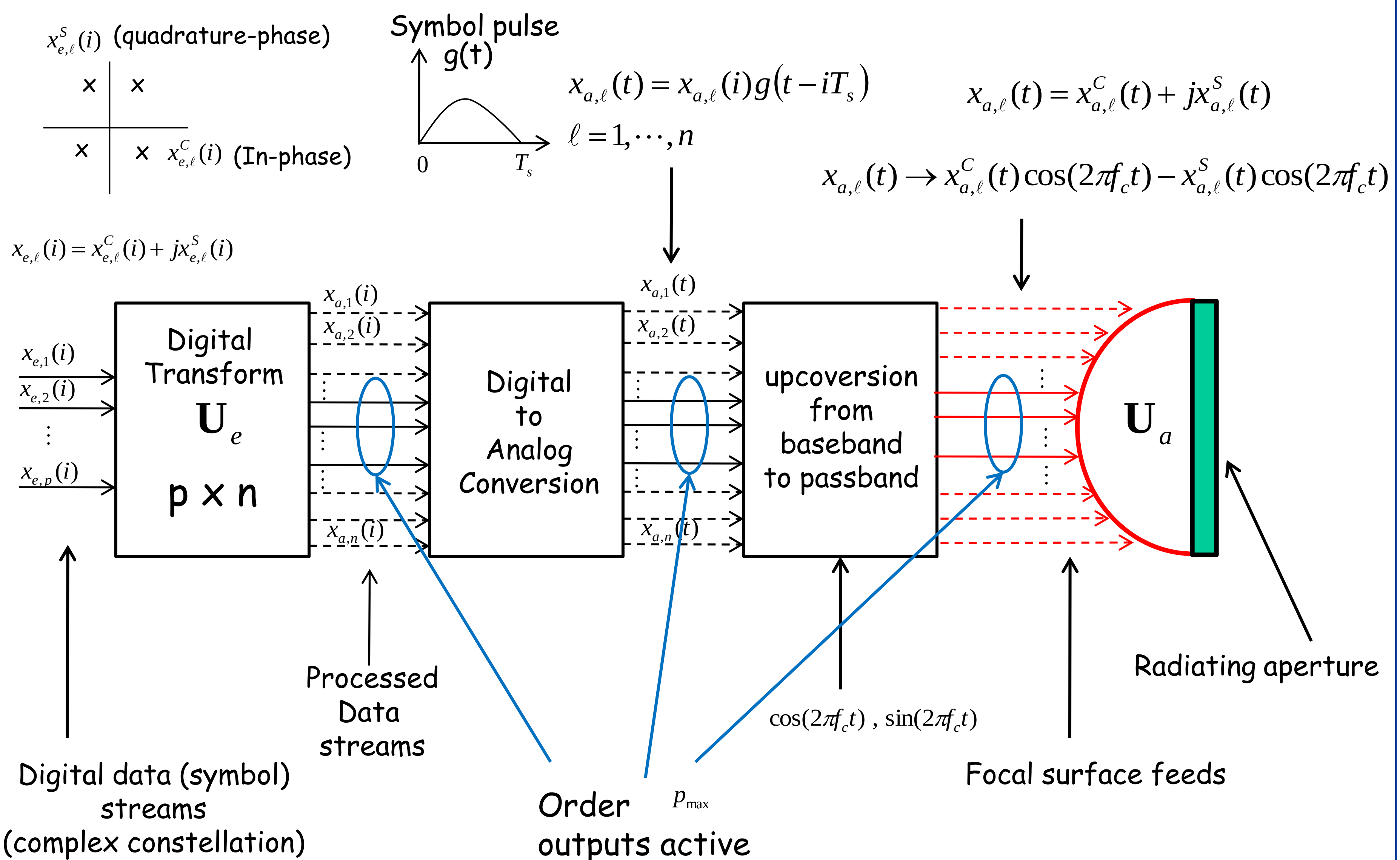
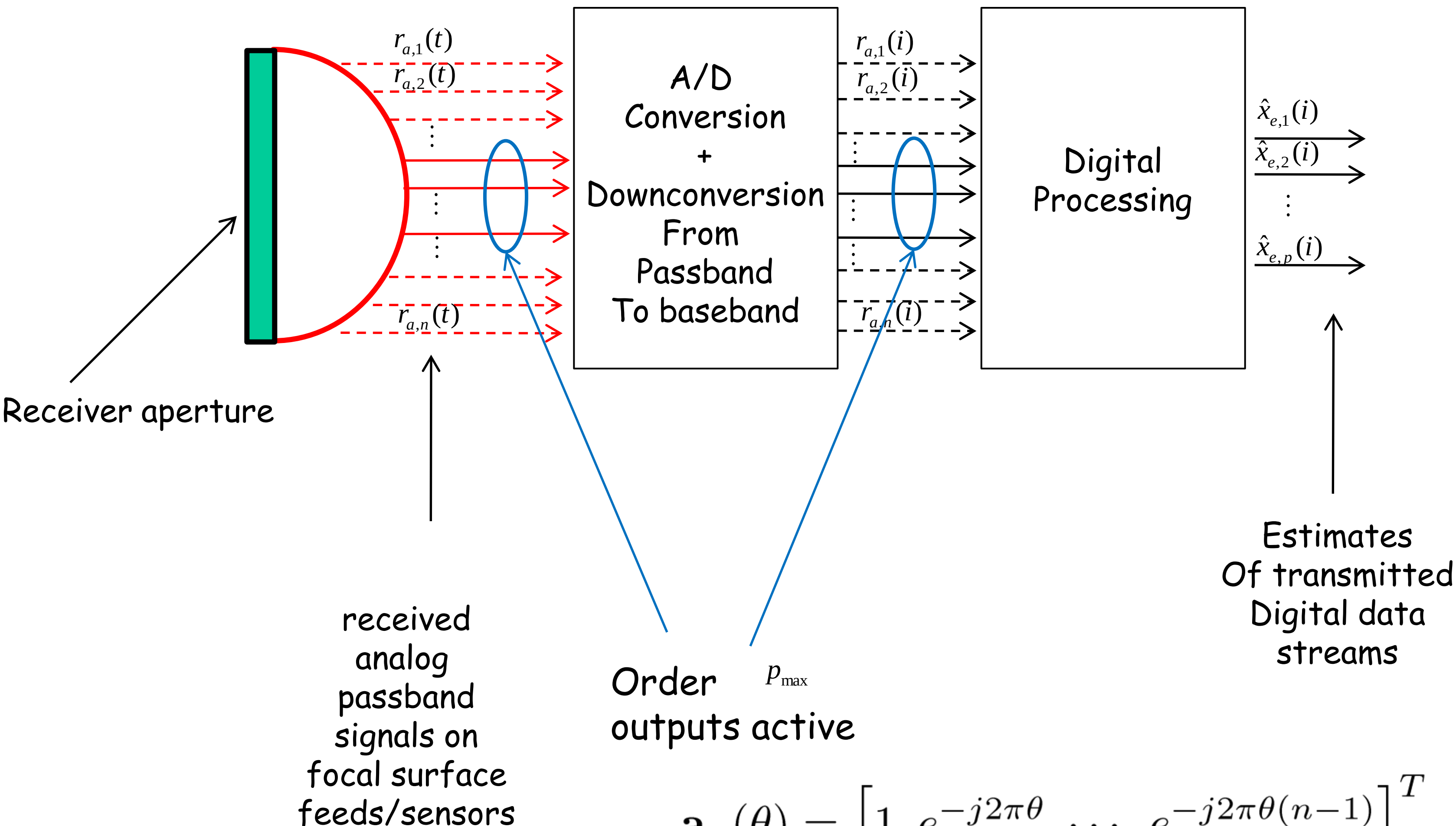
- Ray-tracing design of DLA aperture phase-profile (Fall 2010)
- Detailed full-wave simulation DLA design (Spring 2010)
  - Impact of feeds
  - Far-field beam patterns (at the receiver)
- Prototype building (Spring 2010)
- Prototype measurement (Summer 2010)

Detailed Transmitter Architecture



Detailed Receiver Architecture



$$\mathbf{a}_n(\theta) = \left[1, e^{-j2\pi\theta}, \cdots, e^{-j2\pi\theta(n-1)}\right]^T$$

CAP-MIMO: Overview

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- Combines features from three technologies:
  - Continuous aperture "dish" antennas (power gain)
  - Multi-antenna (MIMO) technology (multiple data streams)
  - Phased arrays (digital beamforming, beam steering)
- Hybrid Analog-Digital Architecture
  - Analog: High-resolution Discrete Lens (Phased) Arrays (DLAs) – Analog spatial beamforming
  - Digital: Discrete Fourier Transform (DFT)!
- Compelling performance gains over state-of-the-art
  - Link capacity
  - Power/bandwidth efficiency
  - A/D complexity

$$\mathbf{a}_n(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi\theta} \\ \vdots \\ e^{-j2\pi\theta(n-1)} \end{bmatrix}^T$$

Transmit-Receive Array Gain

$$\theta = \frac{d}{\lambda_c}\sin(\phi) = 0.5\sin(\phi) \qquad \mathbf{a}_n(\theta) = \left[1, e^{-j2\pi\theta}, \cdots, e^{-j2\pi\theta(n-1)}\right]^T$$

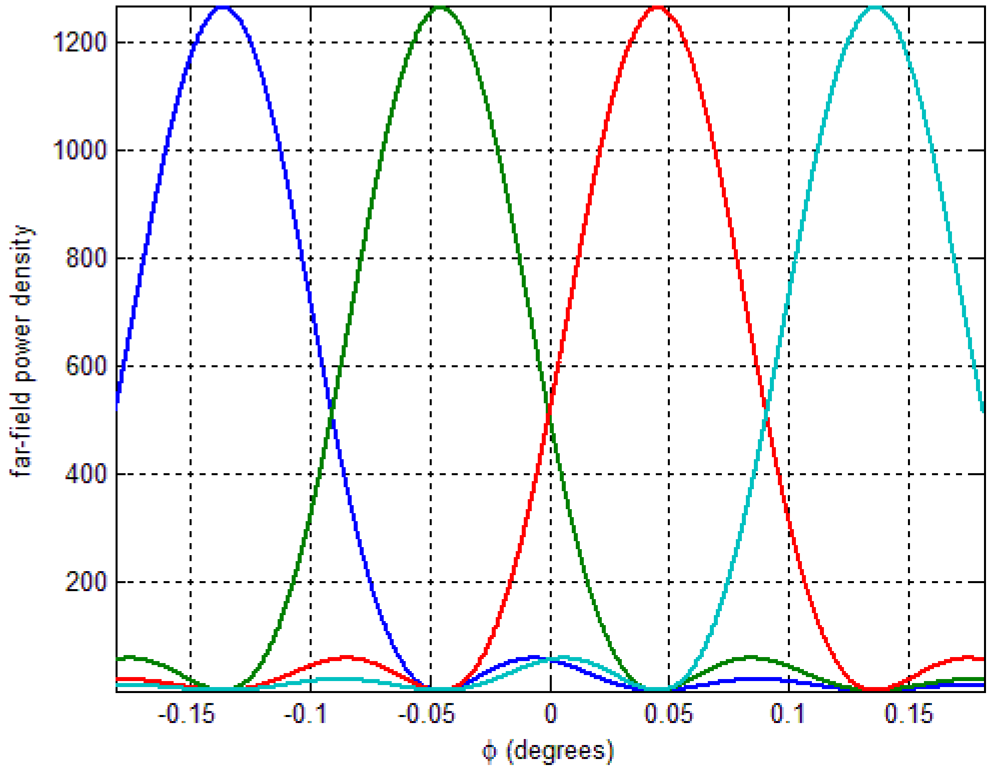
Orthogonal beams: $\theta_i = i\Delta\theta_o = \frac{i}{n}$

$$i = 0, \cdots, n-1$$

Far-field power density: $|b_i(\theta)|^2$

$$b_i(\theta) = \frac{1}{\sqrt{n}}\mathbf{a}_n^H(\theta)\mathbf{a}_n(\theta_i) = \frac{1}{\sqrt{n}}\frac{\sin(\pi n(\theta - \theta_i))}{\sin(\pi(\theta - \theta_i))}$$

TX array gain: n-fold (compared to omni-directional antennas)



$$n = 1245; \;\; p_{max} = 4;$$

Narrow beams: $\Delta\theta_o = \frac{1}{n} \longleftrightarrow \Delta\phi_o = \frac{\lambda_c}{A} \quad \Rightarrow R\Delta\phi_o = \frac{R\lambda_c}{A} = \frac{A}{p_{max}} \Leftrightarrow \boxed{\frac{n}{p_{max}}}$ (Rx gain)