Introduction to radio and radio antennas

The concepts of wave transmission can be more readily grasped if the approach is based upon an understanding of the fundamental circuit elements: inductors and capacitors

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Electromagnetic wave propagation is a simple and beautiful phenomenon, but its rigorous mathematical derivation from Maxwell's equations is formidable. This article makes many of the concepts visible and plausible using only mathematics available to college freshmen and taught today in most high schools. Those who are familiar with the subject will appreciate the simple and direct way in which Dr. Friis' transmission formula is derived. This derivation is new and is published here for the first time.

—David DeWitt, Editor

Transmission lines guide telephone signals, and also power, from one place to another. Although such guidance does not exist for radio signals, it is illustrative to use the transmission of waves between two parallel conductors as an introduction to radio.

A transmission line has a capacitance C and an inductance L, per unit length, and in order to derive the transmission-line equations we must know the fundamental properties of the important circuit elements: inductors and capacitors.

Inductors

Figure 1 shows a long coil of wire with direct current *i*. The resistance of the wire is assumed to be negligible. By means of a magnetic needle indicator it has been found that the magnetic field H inside the coil is constant. Physicists also found for a closed path around currents that the sum of the magnetic fields along the path multiplied by the path lengths is equal to the total current through the area enclosed by the path. Path *ABCDA* in Fig. 1 shows such a path. The magnetic field is zero along this path except along AD, and the total current through the area of the path is $i \times n$. Thus,

Hl = in

Editor's note: Last month an autobiographical memoir by Dr. Friis, *Secenty-Fice Years in an Exciting World*, was published by San Francisco Press, 555 Howard St., San Francisco, Calif.; it also contains "The Wisdom of Harald T. Friis," based on a famous speech (annotated by John R. Pierce) in which the author distills a lifetime's hard-won research experience.



FIGURE 1. Long coil of wire with direct-current flow.

FIGURE 2. Long coil with ac applied voltage.



or, by definition,

$$H = \frac{in}{l}$$
 amperes per meter

In Fig. 2 an ac voltage $v = V_0 \cos 2\pi ft$ is applied to the coil; f is the frequency in hertz and t is the time in seconds. (See Fig. 3 for variation of v vs. $2\pi ft$.) Faraday found that the voltage induced in one turn by i is proportional to the area s of the turn and the rate of change of i or H. Hence,

$$v$$
 (one turn) = $\mu s \frac{dH}{dt} = \mu s \frac{n}{l} \frac{du}{dt}$

where the proportionality factor μ is the permeability of the medium inside the coil (for air, $\mu = 4\pi \times 10^{-7}$ H/m). The voltages in the separate turns are aiding. Therefore, the voltage induced across the coil is

$$v = \mu s \frac{n^2}{l} \frac{di}{dt} = L \frac{di}{dt}$$
 volts (1)

 $L = \mu s(n^2/l)$ is called the inductance of the coil and is measured in henrys.

For $v = V_0 \cos 2\pi ft$, Eq. (1) may be solved by integration to obtain

$$i = \frac{V_0}{2\pi fL} \sin 2\pi ft \text{ amperes}$$
(2)

The values of *i* and *v* are plotted versus $2\pi ft$ in Fig. 3.

Capacitors

Figure 4 shows the plates of a capacitor. Electrostatic experiments showed that the ratio of total charge qon each plate to the voltage v between the plates is constant:

$$\frac{q}{v} = \text{constant} = C$$
 (3)

C is proportional to the ratio of the area s (square



FIGURE 3. Voltage (v) and current (i) vs. 2π ft for inductive circuit of Fig. 2.



FIGURE 4. Basic capacitor configuration.



FIGURE 5. Capacitor with ac applied voltage.

FIGURE 6. Voltage (v) and current (i) vs. 2π ft for capacitive circuit of Fig. 5.



meters) and spacing d (meters) of the plates:

$$C = \epsilon \frac{s}{d} \text{ farads}$$
(3a)

where ϵ , the proportionality factor, is the dielectric constant of the medium between the plates. For free space $\epsilon = (1/36\pi)10^{-9}$ F/m.

In Fig. 5 a voltage $v = V_0 \cos 2\pi ft$ is applied to the plates. The current *i*, called the displacement current, is equal to the rate of change of q. Equation (3) gives

$$i = \frac{dq}{dt} = C \frac{dv}{dt} = -2\pi f C V_0 \sin 2\pi f t \qquad (4)$$

The values of *i* and *v* are plotted versus $2\pi ft$ in Fig. 6.

Transmission lines

Figure 7(A) shows a generator that feeds a long, lossfree uniform line with an inductance of L henries and capacitance of C farads per meter and Fig. 7(B) shows the equivalent lumped-circuit line.

The current i_x through inductor L dx decreases the voltage from AB to CD. Equation (1) gives

 $dv_x = -L \, dx \, \frac{di_x}{dt}$

or

or

$$\frac{dv_x}{dx} = -L\frac{di_x}{dt}$$

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Differentiating with respect to t gives

$$\frac{d^2 v_x}{dx \ dt} = -L \frac{d^2 i_x}{dt^2} \tag{5a}$$

The displacement current through capacitor C dxdecreases the current from A to C. Equation (4) gives

$$di_{x} = -C \, dx \, \frac{dv_{x}}{dt}$$

$$\frac{di_{x}}{dt} = -C \, \frac{dv_{x}}{dt}$$
(6)

dt

dx Differentiating with respect to x gives

$$\frac{d^2 i_x}{dx^2} = -C \frac{d^2 v_x}{dx \, dt} \tag{6a}$$

Substituting the value for $d^2v_x/dx dt$ given by Eq. (5a) gives

$$\frac{d^2 i_x}{dx^2} = LC \frac{d^2 i_x}{dt^2} \tag{7}$$

Similarly, we obtain

$$\frac{d^2 v_x}{dx^2} = LC \frac{d^2 v_x}{dt^2} \tag{8}$$

Mathematicians can solve for i_x and v_x in Eqs. (7) and (8) and get, as one correct solution,

$$i_x = I_0 \sin(2\pi ft - 2\pi fx \sqrt{LC})$$
 (9)

$$v_x = V_0 \sin(2\pi ft - 2\pi fx \sqrt{LC})$$
 (10)

We can verify that these solutions are correct by differentiating i_x in Eq. (9) twice with respect to x and twice with respect to t:

$$\frac{di_x}{dx} = -I_0 \cos \left(2\pi ft - 2\pi fx \sqrt{LC}\right) \times 2\pi f \sqrt{LC}$$

and

$$\frac{d^2 i_x}{dx^2} = -I_0 \sin \left(2\pi f t - 2\pi f x \sqrt{LC}\right) \times (2\pi f \sqrt{LC})^2 \quad (11)$$

Also,

$$\frac{di_x}{dt} = I_0 \cos\left(2\pi ft - 2\pi fx \sqrt{LC}\right) \times 2\pi ft$$

and

$$\frac{d^{2}i_{x}}{dt^{2}} = -I_{0}\sin\left(2\pi ft - 2\pi fx\sqrt{LC}\right) \times (2\pi ft)^{2} \quad (12)$$

Equations (11) and (12) give

$$\frac{d^2 i_x}{dx^2} = LC \frac{d^2 i_x}{dt^2} \tag{13}$$

Equations (7) and (13) are identical. Therefore, Eq. (9) gives a correct solution of (7) and, similarly, (10) gives a correct solution of (8).

We substitute from Eqs. (9) and (10) in (5):

$$-2\pi f \sqrt{LC} V_0 \cos \left(2\pi f t - 2\pi f x \sqrt{LC}\right)$$
$$= -LI_0 2\pi f \cos \left(2\pi f t - 2\pi f x \sqrt{LC}\right)$$

That is,

or

$$V_0 = I_0 \sqrt{\frac{\bar{L}}{C}}$$

 $\sqrt{LC}V_0 = LI_0$

From Eqs. (9) and (10),

$$v_x = i \sqrt{\frac{L}{C}}$$

or the impedance of the line is

$$Z = \frac{v_x}{i_x} = \frac{V_0}{I_0} = \sqrt{\frac{L}{C}} \text{ ohms}$$
(13a)

Values of v_x , as given by Eq. (10), versus x are plotted in Fig. 8 for $t = 0, t = 1/4f, t = 1/2f, \ldots$

Note that the first voltage maximum moves with time from A to B to C to D to E to F with a speed

$$c = \frac{1}{2f\sqrt{LC}} \left/ \frac{1}{2f} = \frac{1}{\sqrt{LC}} \, \mathrm{m/s}$$

or the input generator causes a traveling wave on the line.

The separation between voltage maximums is called the wavelength λ :

$$\lambda = \frac{1}{f\sqrt{LC}} = \frac{c}{f} \text{ meters}$$

Equation (10) shows that the difference in phase ϕ of the voltage at distance x and distance x + d is

$$\phi = 2\pi f(x+d)\sqrt{LC} - 2\pi fx\sqrt{LC}$$
$$= 2\pi fd\sqrt{LC} = 2\pi \frac{d}{\lambda}$$
(14)

Figure 9 shows a uniform strip transmission line in

which the edge effect is neglected. The transmission-line theory gave line impedance $z = \sqrt{L/C}$ ohms.

In this line the inductance L per meter is the inductance of a single-turn coil with area s = 1h and length l = a. Equation (1) gives

$$L = \mu h \frac{1}{a}$$

Similarly, Eq. (3a) gives

$$C = \epsilon \frac{a}{h}$$

The speed of propagation c is given by

$$c = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu}}$$

In free space,

$$\mu = 4\pi \times 10^{-7}$$
 and $\epsilon = \frac{1}{36\pi} \times 10^{-9}$

so $c = 3 \times 10^8$ m/s, which is the speed of light and also the propagation speed for transmission lines constructed



FIGURE 7. A—Long uniform transmission line. B-Equivalent lumped-circuit line.





FIGURE 9. Uniform strip transmission line with traveling wave.



with negligible amounts of insulation and magnetic material.

The line impedance z is given by

$$z = \frac{h}{a}\sqrt{\frac{\mu}{\epsilon}} = 120\pi \frac{h}{a}$$

For h = a = 1 and neglecting edge effects, we get Schelkunoff's formula for the impedance of free space:

$$z_{\rm free \ space} = 120\pi$$
 ohms

A sheet with 120π ohms between two opposite edges of a square of the sheet is called a resistance sheet with 120π ohms per square. It matches waves in free space and, if inserted between the ends of the line, makes a good termination; that is, it causes no reflected waves for the traveling wave on the line when $h < \lambda$. (A conducting sheet $\lambda/4$ behind the resistance sheet is required when h is larger.)

The voltage across the line is hE volts. As shown in Fig. 9, *E* is the maximum value of the electric field in the space between the strips. It is measured in volts per meter. The power absorbed by a circuit of impedance *Z* with an input voltage *E* is $\frac{1}{2}E^2/Z$. The impedance of this transmission line is $Z = 120\pi h/a$. The power flow in the line is, therefore,

$$\frac{1}{2}\frac{hE^2}{z} = \frac{1}{120\pi}\frac{E^2}{ha}$$
 watts

The power flow per unit area or the power intensity in the wave between the strip is then



FIGURE 10. Strip line expanded.

FIGURE 11. Strip line feeding a paraboloid.



$$\frac{\frac{1}{2}E^2}{120\pi}$$
 watts (same for radio waves) (14a)

Note that all the power flows in the air between the strip conductors. This is true for all transmission lines. The conductors act only as guides for the waves.

Figure 9 also shows the magnetic field vector H. Its magnitude is found by choosing a path *ABCDA* and adding the field times path length as explained in the first section on inductors. The current through the areas of this path is i and the field is zero except along path *AD*. Hence,

$$Ha = i$$

Equation (13a) gives

$$i = \frac{v}{\sqrt{L/C}} = \frac{Eh}{120\pi h/a} = \frac{Ea}{120\pi}$$

Therefore, $H = \frac{i}{a} = \frac{E}{120\pi}$

Figure 9 shows the power intensity vector P, also called the Poynting vector, from (14a),

$$P = \frac{\frac{1}{2}E^2}{120\pi} = \frac{1}{2}EH$$
 watts per square meter

Recapitulating what happens in the idealized line illustrated in Fig. 9, it has been shown (1) that the available power from the generator has been changed to a plane electromagnetic wave that moves with the speed of light toward the termination of the line, (2) that the power flow intensity is constant over the cross section of the wave and equal to $\frac{1}{2}EH$ watt where E is the electric field and H the magnetic field of the wave, and (3) that the impedance of the line is $120\pi h/a$ ohms and the free-space medium between the strips is 120π ohms.

The strip-line dimensions of width a and separation h may be increased gradually as shown in Fig. 10. The traveling wave will continue through this expanded section and produce a large radiating wavefront at the opening *ABCD*. We have now a radio transmitting antenna (Schelkunoff's paddle antenna). Alternatively, the strip line may feed a paraboloid, as shown in Fig. 11. We have, in other words, illustrated the passing of guided waves to radio waves; we shall next discuss some important properties of radio antennas and radio transmission.

Effective area of an antenna. The effective area A_{eff} of an antenna is defined as shown in Fig. 12, for the case of receiving, as the available power P_R from the antenna





Incoming wave with power density P_D

divided by the power density P_D of the incoming wave:

$$A_{\rm eff} = \frac{P_R}{P_D}$$

If the illumination of the antenna when used for transmitting is uniform (that is, if the electromagnetic field is constant and in phase over the plane of the aperture and zero outside), the effective area is by reciprocity equal to the geometrical area of the aperture.

Examples: The effective area of the paddle antenna shown in Fig. 10 is approximately equal to the opening area *ABCD* and the effective area of the paraboloid antenna shown in Fig. 11 is about two thirds of the opening area of the paraboloid. The effective area of a small loss-free dipole, which will be calculated later, is $(\lambda/2) \times (\lambda/4)$. As usual, all linear dimensions are in meters.

Propagation loss between two antennas in free space. That light propagates as a wave was suggested some 300 years ago by Huygens, and resulted in Huygens' principle: "Every part of a wavefront can be regarded as a source of disturbance that emits a spherical wavelet." Fresnel made his great contribution some 150 years later by realizing that the relative phase ϕ between two wavelets can be calculated from their path difference *d* and wavelength λ [compare with Eq. (14)]:

$$\phi = 2\pi \frac{a}{\lambda}$$

Figure 13 shows how Fresnel graphically found the field due to the wavelets in a plane wave at a distant point. Figure 13(B), a side view of the wave, indicates the wavelets on the wavefront that radiate spherically toward the distant point *P*. Figure 13(A) shows how the wavefront is divided into equal-area rings 1, 2, 3, \cdots . The Huygens sources in each ring have the same distance to point *P* and are therefore in phase at *P*. Each of the rings has the same area or the same number of wavelets and therefore produces the same fields E_1, E_2, E_3, \cdots at *P* when the angle ϕ is small.

Figure 13(C) shows how two parallel fields

$$E_1 = E \cos\left(2\pi ft - \phi_1\right)$$

and

$$E_2 = E \cos\left(2\pi ft - \phi_2\right)$$

can be added graphically by plotting OA = E first and then AB = E so that the angle between AB and the direction of OA is $\phi_2 - \phi_1$. The sum of E_1 and E_2 is then OB. That this is correct can be shown as follows:

$$E_1 + E_2 = E \cos (2\pi f t - \phi_1) + E \cos (2\pi f t - \phi_2)$$
$$= E[\cos (2\pi f t - \phi_1) + \cos (2\pi f t - \phi_2)]$$

The formula for the addition of two cosine functions gives

$$E_1 + E_2 = 2E \cos\left(2\pi ft - \frac{\phi_1 + \phi_2}{2}\right) \cos \frac{\phi_2 - \phi_1}{2}$$

or the amplitude of $E_1 + E_2$ is

$$2E\cos{\frac{\phi_2-\phi}{2}}$$

and this is the length of OB in Fig. 13(C).

The fields at point P from the wavelets can now be added graphically, as shown in Fig. 13(D). Starting at point O, the field E_1 is plotted first with arbitrary length and direction and it is followed by E_2 so that E_1 and E_2 form an angle

$$\Delta \phi = 2\pi \; \frac{d_2 - d}{\lambda}$$

where d_2 is the distance from the sources in ring 2 and dthe distance from the sources in the center element 1. The fields E_3 , E_4 , \cdots are plotted by continuing in the same manner. The small phase angles $\Delta \phi$ are alike when angle ϕ is small and the ends of the fields therefore follow a circle. This circle turns into a spiral when angle ϕ increases and finally gives the original field E of the wave when the rings expand to infinity.

The phase of the field from the ring with radius r with respect to field E_1 from the field from the center element is

$$\phi = \frac{2\pi(d'-d)}{\lambda} \approx \frac{\pi r^2}{\lambda d}$$

Fresnel could have found the field $E_r = E_1 + E_2 + E_3 + E_4 + E_5$ due to all the rings with radii less than r. Figure 13(D) shows that it is

$$E_r = E\phi = E \frac{\pi r^2}{\lambda d}$$
 volts

FIGURE 13. Propagation of light waves. A—Front view of wave. B—Side view of wave. C—Addition of two fields. D—Addition of wavelet fields.



$$\frac{E_r}{E}=\frac{\pi r^2}{\lambda d}^*$$

This important formula will now be used to find the ratio of the power received by a light absorber with

* Since E_r (which is the graphical sum of E_1 , E_2 , E_3 , E_4 , and E_5) approaches $5E_1$ for very small values of ϕ , E_1 must be proportional to $1/\lambda$; hence the field of a Huygens source is proportional to $1/\lambda$, another very useful conclusion.





₽_R,



Isotropic transmitting antenna

FIGURE 15. Relation between the gain and the effective area of an antenna.

FIGURE 16. Gain of a small dipole.



area A_R and the power transmitted by a light transmitter with area $A_T = \pi r^2$. Figure 14 shows the light circuit. The received and transmitted power flows per unit area are proportional to the square of the light voltages. Hence.

$$\frac{P_R}{P_T} = \left(\frac{E_R}{E}\right)^2 \frac{A_R}{A_T} = \left(\frac{\pi r^2}{\lambda d}\right)^2 \frac{A_R}{A_T} = \frac{A_T A_R}{\lambda^2 d^2} \qquad (15)$$

Since light waves and radio waves obey the same laws, A_T and A_R may be considered to be the effective areas of radio transmitting and receiving antennas; this formula is identical to my transmission formula for a radio circuit¹ and it is interesting that it could have been derived long before Maxwell published his theory of electromagnetic waves in 1864 and Hertz discovered radio waves in 1888. Also, the formula applies to acoustic waves. Hence, there should be a derivation independent of electromagnetic theory.

Relation between the gain and effective area of an antenna. Figure 15 shows a hypothetical isotropic antenna that radiates a uniform field in all directions. The power flow at distance d is equal to the transmitted power P_T divided by $4\pi d^2$, the area of a surrounding sphere, and the received power is

$$P_R = P_T \times \frac{1}{4\pi d^2} \times A_R$$
 watts

Replacing the isotropic antenna with a transmitting antenna of area A_T and using Eq. (15), we have, for the received power,

$$P_R' = P_T \frac{A_T A_R}{d^2 \lambda^2}$$
 watts

The gain g of the new transmitting antenna, defined as P_R'/P_R , is

$$g = 4\pi \frac{A_T}{\lambda^2} \tag{16}$$

and this important formula applies to receiving antennas as well.

The gain of a small dipole. A wire carrying an alternating current radiates power. Hertz showed this experimentally years ago and later work showed that the radiation field in a direction is proportional to the projected length of the wire in that direction. Figure 16 shows the radiation patterns of a current element or small dipole (cross section of two circles with radii E/2) and an iso-







FIGURE 18. The distant field E from a small dipole with uniform current I.

tropic antenna (cross section of a circle with radius *E*). *E* is the field at a large distance *R*. The broken-line circle with radius *R* is the cross section of a sphere. The surface of the sphere is divided into circular rings with area $2\pi R \sin \theta \times R \, d\theta$. The power flow through the ring due to the dipole is

$$dP_{\text{dipole}} = \frac{\frac{1}{2} (E \sin \theta)^2}{120\pi} \times 2\pi R \sin \theta \times R \, d\theta$$
$$= \frac{\frac{1}{2} E^2}{120\pi} \times 2\pi R^2 \sin^3 \theta \, d\theta$$

The power radiated by the dipole is then

$$P_{\rm dipole} = \frac{\frac{1}{2}E^2}{120\pi} 2\pi R^2 \times \int_0^\pi \sin^3\theta \ d\theta = \frac{\frac{1}{2}E^2}{120\pi} 2\pi R^2 \times \frac{4}{3}$$

The power radiated by the isotropic antenna is

$$P_{\rm iso} = \frac{\frac{1}{2}E^2}{120\pi} 4\pi R^2$$

The gain of the dipole is then

$$g_{\rm di} = \frac{P_{\rm iso}}{P_{\rm di}} = 1.5$$
 (17)

Effective area of a small dipole (Fig. 17). Equations (16) and (17) give

$$A_{\rm di} = g_{\rm di} \times \frac{\lambda^2}{4\pi} = 1.5 \times \frac{\lambda^2}{4\pi} \approx \frac{\lambda}{2} \times \frac{\lambda}{4}$$
 square meters (18)

Note that this area is independent of the dipole length h (which is always small compared with the wavelength).

Radiation resistance of a small dipole. A small dipole with alternating current radiates power and, therefore, it has a resistance called the radiation resistance.

The power received by the dipole shown in Fig. 17 is

$$P_{\rm di} = \frac{\frac{1}{2}E^2}{120\pi} A_{\rm di} = \frac{\frac{1}{2}E^2}{120\pi} \times 1.5 \times \frac{\lambda^2}{4\pi} \text{ watts}$$
 (19)

The dipole is equivalent to a generator with resistance R_{ra} and induced voltages $E \times h$; the available power is

$$P_{\rm di} = \frac{1/2(Eh)^2}{4R_{\rm ra}}$$
 watts (20)

Equations (19) and (20) give

$$R_{\rm ra} = 80\pi^2 \left(\frac{h}{\lambda}\right)^2$$
 ohms (21)

Note that the radiation resistance is small when h/λ is small, and this makes it difficult to construct an efficient matching circuit to an output line.

The distant field from a small dipole with uniform

current. For the circuit shown in Fig. 18, Eq. (15) gives

$$P_R = P_T \frac{A_{\rm di}A_R}{d^2\lambda^2}$$
 watts (22)

We also have

$$P_R = \frac{\frac{1}{2}E^2}{120\pi} A_R$$
 watts (23)

and, since I is maximum value of the alternating current,

$$P_T = \frac{1}{2}R_a I^2 \text{ watts}$$
 (24)

Equations (18) and (21)-(24) give

1

$$E = 60\pi \frac{h}{d\lambda} I$$
 volts per meter

Summary

Assuming unobstructed free-space transmission and using for the most part only mathematics taught in many high schools, it has been possible to derive the fundamental properties of radio antennas and radio transmission.

Dr. J. R. Pierce got me started on this article in the fall of 1969 by encouraging his co-workers to find a simple derivation of my old transmission formula. One of these co-workers, C. L. Ruthroff, mentioned the old work on light waves by Huygens and Fresnel and this set me off in the right direction. Dr. S. A. Schelkunoff suggested that I simplify the discussion by deleting the complex algebra in the original version. Lou Mitchel, physics teacher at Rumson High School, read the paper with one of his students and recommended its publication.

REFERENCE

1. Friis, H. T., "A note on a simple transmission formula," Proc. IRE, vol. 34, pp. 254-256, May 1946.

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Harald Trap Friis (F) was born in Naestved, Denmark, in 1893. He received the electrical engineer and doctor of science degrees from the Royal Technical College, Copenhagen, in 1916 and 1938, respectively. In 1919 he moved to the United States and, after a period of study at Columbia University, he joined the Western Electric Company's Research Department, which was later to become Bell Telephone Laboratories, Inc. He was made director of radio research in 1945. In 1952 he became director of research in high frequency and electronics. During his career with the Bell System he contributed substantially to almost every aspect of the radio art, including vacuum tubes, the design of the first commercial superheterodyne receiver, noise, antennas and propagation, radar, and microwaves. Following his retirement from Bell Laboratories in 1958, he served until 1968 as consultant to the Hewlett-Packard Company. At present he is doing consulting work from his home at Rumson, N.J.

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