Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas

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***Abstract*—A cellular base station serves a multiplicity of single-antenna terminals over the same time-frequency interval. Time-division duplex operation combined with reverse-link pilots enables the base station to estimate the reciprocal forward- and reverse-link channels. The conjugate-transpose of the channel estimates are used as a linear precoder and combiner respectively on the forward and reverse links. Propagation, unknown to both terminals and base station, comprises fast fading, log-normal shadow fading, and geometric attenuation. In the limit of an infinite number of antennas a complete multi-cellular analysis, which accounts for inter-cellular interference and the overhead and errors associated with channel-state information, yields a number of mathematically exact conclusions and points to a desirable direction towards which cellular wireless could evolve. In particular the effects of uncorrelated noise and fast fading vanish, throughput and the number of terminals are independent of the size of the cells, spectral efficiency is independent of bandwidth, and the required transmitted energy per bit vanishes. The only remaining impairment is inter-cellular interference caused by re-use of the pilot sequences in other cells (pilot contamination) which does not vanish with unlimited number of antennas.**

***Index Terms*—Multiuser MIMO, pilot contamination, noncooperative cellular wireless, active antenna arrays.**

I. Introduction

MULTIPLE-ANTENNA (conveniently referred to as MIMO - *Multiple-Input, Multiple-Output*) technology is a key feature of all advanced cellular wireless systems [1], but it has yet to be adopted on a scale commensurate with its true potential. There are several reasons for this. Cheaper alternatives to increasing throughput, such as purchasing more spectrum, are invariably adopted before more expensive and technologically sophisticated solutions. A point-to-point MIMO system [2] requires expensive multiple-antenna terminals. Multiplexing gains may disappear near the edges of the cell where signal levels are low relative to interference or in a propagation environment which is insufficiently dominated by scattering.

An alternative to a point-to-point MIMO system is a multiuser MIMO system [3], [4], [5], [6] in which an antenna array simultaneously serves a multiplicity of autonomous terminals. These terminals can be cheap, single-antenna devices, and the multiplexing throughput gains are shared among the terminals. A multi-user MIMO system is more tolerant of the propagation environment than a point-to-point system: under line-of-sight propagation conditions multiplexing gains can disappear for a point-to-point system, but are retained in the multi-user system provided the angular separation of the terminals exceeds the Rayleigh resolution of the array.

Channel-state information (CSI) plays a key role in a multi-user MIMO system. Forward-link data transmission requires that the base station know the forward channel, and reverse-link data transmission requires that the base station know the reverse channel.

A. Multi-user MIMO systems with very large antenna arrays

Multi-user MIMO operation with a large excess of base station antennas compared with terminals was advocated in [7] which considers a single-cell time-division duplex (TDD) scenario in which a time-slot over which the channel can be assumed constant is divided between reverse-link pilots and forward-link data transmission. The pilots, through reciprocity, provide the base station with an estimate of the forward channel, which in turn generates a linear pre-coder for data transmission. The time required for pilots is proportional to the number of terminals served and is independent of the number of base station antennas [8]. Irrespective of the number of base station antennas, the number of terminals that can be served is therefore limited by the coherence time, which itself depends on the mobility of the terminals. The principal finding of [7] is that, even with a very noisy channel estimate, the addition of more base station antennas is always beneficial, and in the limit of an infinite number of antennas, the effects of fast fading and uncorrelated noise vanish. One can always recover from low SNR conditions by adding a sufficient number of antennas.

The present paper considers multi-user MIMO operation with an infinite number of base station antennas in a multi-cellular environment. A new phenomenon emerges which was not encountered in the single-cell scenario of [7]: *pilot contamination* [9]. The same band of frequencies is re-used with factor one, three, or seven among the cells. Of necessity, the same orthogonal pilot sequences are re-used - possibly multiplied by an orthogonal transformation - among the cells. In the course of learning the channels to its own terminals, a base station inadvertently learns the channel to terminals in other cells who share the same pilot sequence, or whose pilot sequences are merely correlated with the pilot sequences of its own terminals. While transmitting data to its terminals the base station is also selectively transmitting data to terminals in other cells. Similarly when the base station combines its reverse-link signals to receive the individual data transmissions of its terminals, it is also coherently combining signals from terminals in other cells. The resulting inter-cellular interference persists even with an infinite number of antennas.

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Pilot contamination is a fundamental problem which is easy to overlook if one gratuitously assumes that channel-state information is available for free. One of the strengths of the present research is that the acquisition of CSI is treated as a central activity.

B. Propagation and multiuser MIMO operation

The distinguishing features of the present paper are twofold: first, the MIMO operation is multiuser rather than point-to-point, and second, an unlimited number of base station antennas serves a fixed number of single-antenna terminals. These two conditions enable us to escape limitations which the typical propagation environment would otherwise impose.

The case of large numbers of antennas in the context of point-to-point MIMO operation is considered, for example, in [10], [11] and [12]. These papers contrast a naive model where the propagation coefficients are uncorrelated over space (which promises, for sufficiently high signal-to-noise ratios, a capacity which grows linearly with the smaller of the number of transmit or receive antennas) and physically realistic models where the propagation coefficients are correlated over space (where the capacity grows at a sub-linear rate). However, for the multiuser MIMO operation considered in this paper the single-antenna terminals are distributed randomly over the cell, and they are typically separated by hundreds of wavelengths or more.

Thus, under the propagation models in the papers cited above, the propagation vectors between the base station array and different terminals would be uncorrelated. In fact the multiuser MIMO results of this paper would hold also under line-of-sight propagation conditions because, for a sufficiently large base station array, the typical angular spacing between any two terminals would be greater than the angular Rayleigh resolution of the array, hence the propagation vectors for different terminals become asymptotically orthogonal. Consider, for example, a linear array of antennas with half-wavelength spacing in a line-of-sight environment. The propagation vector for a terminal in the far-field region of the array is $\exp(i\pi um),\ m = 1, \cdots, M$, where u is the sine of the angle of the terminal with respect to the perpendicular to the array. If u is randomly distributed uniformly over the interval $[-1, 1]$, then it can be shown that the inner product between the propagation vectors of any two terminals has a standard deviation of $\sqrt{M}$ which is the same as for independent Rayleigh fading.

A wireless network is considered in [13]. The propagation medium is modeled as two-dimensional, as a result of which a half-wavelength spaced regular rectangular array can only access a number of propagation degrees-of-freedom which grows as the square-root of the number of antennas. Consequently a dense wireless network has a sum-throughput which can only grow proportional to the square-root of the number of nodes. In contrast the multiuser MIMO scenario of this paper has an unlimited number of base station antennas (which could be arranged as a ring-array with half-wavelength circumferential spacing) serving a fixed number of terminals: there is no attempt to serve an increasing number of terminals as the number of base station antennas increases. In fact the maximum number of terminals which can be served is limited by the time that it takes to acquire CSI from the moving terminals. As we show later, operating with a large excess of base station antennas compared with the number of terminals is a desirable condition.

The critical assumption which our analysis makes about propagation is that, as the number of base station antennas grows, the inner products between propagation vectors for different terminals grow at lesser rates than the inner products of propagation vectors with themselves. This condition holds under the propagation models cited above, and it also holds under line-of-sight conditions. This assumption would be incorrect if, for example, the terminals were located in a waveguide which had fewer normal modes than the number of terminals.

C. Approach and summary of results

We consider a cellular system consisting of noncooperative hexagonal cells with frequency re-use of one, three, or seven, TDD operation, Orthogonal Frequency Division Multiplexing (OFDM), and base station arrays comprising M antennas where $M \to \infty$, where each base station serves K single-antenna terminals. The terminals are located uniform-randomly (with the exclusion of a disk centered on the base station) in the cell which services them. The propagation is a combination of fast fading (which changes over a scale of wavelength) and slow fading (log-normal and geometric decay). Neither the base station nor the terminals have prior knowledge of the channels, and all CSI is acquired from reverse-link pilots which must be scheduled, along with forward- and reverse-link data transmissions, in a coherence interval of some specified duration. Within a cell every terminal is assigned an orthogonal time-frequency pilot sequence. These same pilot sequences are re-used in other cells according to the frequency re-use factor. The base station correlates the received pilot transmissions - which are corrupted by pilot transmissions from other cells - to produce its channel estimates. We assume that all transmissions and receptions are synchronous (arguing later that, from the standpoint of pilot contamination, this constitutes the worst possible case). There is no cooperation or sharing of information among cells, and there is no power control. Forward- and reverse- multiuser MIMO transmission is employed, with the simplest sort of linear pre-coding and combining. On the forward link the base station uses a linear pre-coder which is a scaled version of the conjugate transpose of the forward channel estimate, and on the reverse link the base station combines its received antenna signals by multiplication by the conjugate transpose of the reverse channel estimate.

In our analysis we let the number of antennas, M, grow without limit. We assume that the M-component fast-propagation vector between any terminal and any base station array has an L2-norm which grows as M, while the inner product between any two different propagation vectors grows more slowly to conclude that the effects of additive receiver noise and fast fading disappear, as does intra-cellular interference. The only remaining impediment is inter-cellular interference from transmissions which are associated with the same pilot sequence. We obtain simple expressions for the signal-to-interference ratio (SIR) which are random due to their dependence on the slow fading, and which are equal for all OFDM tones. We numerically determine the cumulative distribution functions of the forward- and reverse-SIRs. We translate the SIR expressions into capacities by assuming that Gaussian signaling is employed, and by treating the inter-cellular interference as noise. We are interested in the mean-throughput per cell, the number of terminals which can be advantageously served in each cell, the mean-throughput per terminal, and the .95-likely throughput per terminal. Our quantitative results depend on only a few modeling parameters: the log-normal shadow fading standard deviation, the geometric attenuation exponent, and the ratio of the radius of the disk from which terminals are excluded to the radius of the cell.

In addition to the numerically-derived conclusions, we obtain several mathematically-exact conclusions (subject, of course, to the limitations of the model): the throughput per cell and the number of terminals per cell are independent of the size of the cell, the spectral efficiency is independent of the bandwidth, and the required transmitted energy per bit vanishes.

Several approximate conclusions hold: the optimum number of terminals to serve (from the standpoint of maximizing the mean throughput) is equal to one-half of the duration of the coherence interval divided by the delay-spread, the throughput per terminal is independent of the coherence time, the effect of doubling the coherence time is to permit twice as many terminals to be serviced, and the reverse-link performance is nearly identical to the forward-link performance, although the statistics of the SIRs are slightly different.

For a scenario in which the coherence time is 500 microseconds (which could accommodate TGV - *Train à Grande Vitesse* - speeds), a delay spread of 4.8 microseconds, and a bandwidth of 20 megahertz, with frequency re-use of seven the forward link has the following performance: each cell can serve 42 terminals, the mean net throughput per terminal is 17 megabits/sec, the 95%-likely net throughput per terminal is 3.6 megabits/sec, and the mean net throughput per cell is 730 megabits/sec (equivalent to a spectral efficiency of 36.5 bits/sec/Hz). More aggressive frequency reuse (factors of three or one) increase the mean-throughput, but decrease the 95%-likely throughput.

D. Outline of Paper

Section II describes the multi-cell scenario and propagation model. Section III discusses the reverse-link pilots. Sections IV and V analyze the multiuser reverse and forward data transmission as the number of base station antennas becomes infinite. The only remaining impairment is inter-cell interference due to pilot contamination. The multi-cell analysis is particularly simple, and certain parameters, including the absolute transmit powers and the absolute size of the cells, disappear from the formulation. This analysis produces closed-form expressions for effective signal-to-interference ratios (SIRs) which depend only on the random positions of the terminals and shadow-fading coefficients. In turn the SIRs translate directly into capacity expressions. Section VI numerically obtains cumulative distribution functions for the SIRs and capacities for a particular scenario. Section VII discusses the ramifications of our results.

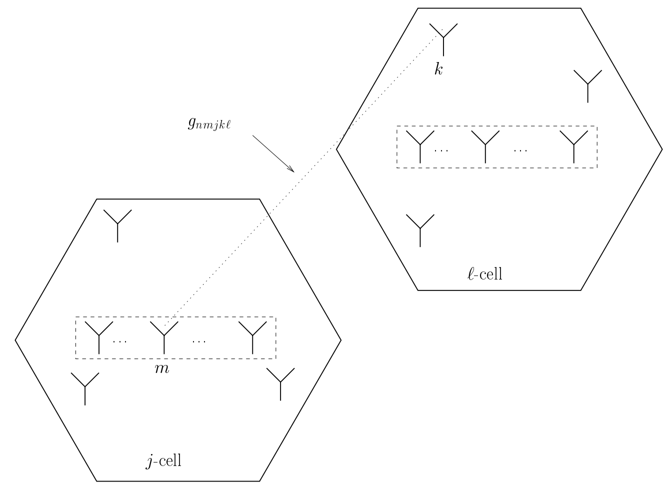

Fig. 1. The propagation coefficient between the k-th terminal in the ℓ-th cell, and the m-th base station antenna of the j-th cell, in the n-th subcarrier, is denoted by $g_{nmjk\ell}$.

II. SCENARIO

Our scenario entails a hexagonal cellular geometry, base stations having an unlimited number of antennas, terminals having single antennas, OFDM, time-division duplex (TDD) operation, and fast fading upon which is superimposed geometric attenuation and log-normal shadow fading.

A. Hexagonal cells

The cells are hexagonal with a radius (from center to vertex) of $r_{\rm c}$. Within each cell, K terminals are placed randomly, uniformly distributed over the cell, with the exclusion of a central disk of radius $r_{\rm h}$. At the center of the cell is a base station array comprising M omnidirectional antennas, where in the subsequent analysis, M grows without limit.

B. OFDM

We assume that OFDM is utilized. We denote the OFDM symbol interval by $T_{\rm s}$, the subcarrier spacing by Δf, the useful symbol duration by $T_{\rm u} = 1/\Delta f$, and the guard interval (duration of the cyclic prefix) by $T_{\rm g} = T_{\rm s} - T_{\rm u}$. We call the reciprocal of the guard interval, when measured in subcarrier spacings, the "frequency smoothness interval",

$$N_{\rm smooth} = 1/(T_{\rm g}\Delta f). \tag{1}$$

C. Propagation

For our subsequent analysis we need to describe the propagation coefficient between a single-antenna terminal in one cell, and a base station antenna in another cell. Because of TDD operation and reciprocity the propagation is the same for either a downlink or an uplink transmission.

As shown in Fig. 1, we denote the complex propagation coefficient between the m-th base station antenna in the j-th cell, and the k-th terminal in the ℓ-th cell in the n-th subcarrier by $g_{nmjk\ell}$ which, in turn, is equal to a complex fast fading factor times an amplitude factor that accounts for geometric attenuation and shadow fading,

$$\begin{aligned} g_{nmjk\ell} &= h_{nmjk\ell} \cdot \beta_{jk\ell}^{1/2}, \\ & \quad n = 1, \cdots, N_{\mathrm{FFT}},\ m = 1, \cdots, M, \\ & \quad j = 1, \cdots, L,\ k = 1, \cdots, K, \\ & \quad \ell = 1, \cdots, L, \end{aligned} \tag{2}$$

where N_{FFT} is the number of subcarriers, M is the number of base station antennas in each cell, L is the number of active cells (i.e., re-using the same band of frequencies), and K is the number of terminals in each cell. The fast fading coefficients, $h_{nmjk\ell}$, are assumed to be zero-mean and unit-variance. With respect to the frequency index, n, the fast fading is assumed to be piecewise-constant over N_{smooth} successive subcarriers, where N_{smooth} is the frequency smoothness interval (1). Only one pilot symbol per smoothness interval is required. The second factor in (2) is assumed constant with respect to both frequency and with respect to the index of the base station antenna since the geometric and shadow fading change slowly over space, and it factors as follows:

$$\beta_{jk\ell} = \frac{z_{jk\ell}}{r_{jk\ell}^{\gamma}}. \tag{3}$$

Here $r_{jk\ell}$ is the distance between the k-th terminal in the ℓ-th cell and the base station in the j-th cell, γ is the decay exponent, and $z_{jk\ell}$ is a log-normal random variable, i.e., the quantity $10\log_{10}(z_{jk\ell})$ is distributed zero-mean Gaussian with a standard deviation of σ_{shad}. The shadow fading $\{z_{jk\ell}\}$ is statistically independent over all three indices. The ranges $\{r_{jk\ell}\}$ are statistically independent over k and ℓ, but statistically dependent over j, because the only randomness that affects $r_{j_1k\ell}$ and $r_{j_2k\ell}$ is the position of the k-th terminal in the ℓ-th cell.

Throughout we assume that both the terminals and the base station are ignorant of the propagation coefficients.

III. Reverse-Link Pilots

Reverse-link pilots are required for both forward and reverse data transmission. A total of τ OFDM symbols are used entirely for pilots. The remainder of the coherence interval is used for transmitting data, either on the forward link or the reverse link or both.

A. Maximum number of terminals

If the channel response changed arbitrarily fast with frequency then, over τ OFDM symbols the base station could only learn the channel for τ terminals in its cell. In general the channel response is constant over N_{smooth} consecutive subcarriers and the base station can learn the channel for a total of $K_{\max} = \tau N_{\mathrm{smooth}}$ terminals. This number has a simple interpretation in the time domain. The guard interval T_{g} is chosen to be greater than the largest possible delay-spread, T_{d}; assume that $T_{\mathrm{d}} = T_{\mathrm{g}}$. Then, according to (1), the maximum number of terminals is

$$\begin{aligned} K_{\max} &= \tau N_{\mathrm{smooth}} \\ &= \frac{\tau}{T_{\mathrm{d}}\Delta f} = \frac{(\tau T_{\mathrm{s}})T_{\mathrm{u}}}{T_{\mathrm{d}}T_{\mathrm{s}}} = \left(\frac{T_{\mathrm{pilot}}}{T_{\mathrm{d}}}\right)\left(\frac{T_{\mathrm{u}}}{T_{\mathrm{s}}}\right), \end{aligned} \tag{4}$$

where $T_{\mathrm{pilot}} = \tau T_{\mathrm{s}}$ is the time spent on sending reverse pilots. Training could be accomplished directly in the time-domain (i.e., without OFDM) by transmitting impulses from a succession of different terminals spaced by the delay-spread. The factor $T_{\mathrm{u}}/T_{\mathrm{s}}$ reflects the inefficiency of OFDM due to the cyclic prefix. Another interpretation for the frequency smoothness interval is that the quantity $N_{\mathrm{smooth}}\Delta f$ is the Nyquist sampling interval in frequency for the time-limited channel impulse response.

The simplest way to send reverse-link pilots would be to assign each terminal one unique time-frequency index for its pilot (e.g., one subcarrier within each smoothness interval and within one OFDM symbol). More generally one could assign mutually orthogonal sequences of time-frequency pilots to the terminals as discussed later in VII-F.

B. Pilot contamination

The same band of frequencies is shared by a multiplicity of cells. If each cell is serving the maximum number of terminals (4) then the pilot signals received by a base station are contaminated by pilots transmitted by terminals in other cells.

We assume that a total of L base stations share the same band of frequencies and the same set of K pilot signals. Furthermore we assume synchronized transmissions and reception. We argue later that synchronized transmission constitutes a worse-case scenario from the standpoint of pilot contamination.

After any required processing each base station obtains an estimate for the propagation between its antennas and its terminals which is contaminated by propagation from terminals in others cells. Let $\hat{G}_{jj}$ denote the estimate for the $M \times K$ propagation matrix between the M base station antennas of the j-th cell, and the K terminals in the j-th cell; for notational simplicity we suppress the dependence of $\hat{G}_{jj}$ on the sub-carrier index:

$$\hat{G}_{jj} = \sqrt{\rho_{\mathrm{p}}} \sum_{\ell=1}^{L} G_{j\ell} + V_j, \tag{5}$$

where $G_{j\ell}$ is the $M \times K$ propagation matrix between the K terminals in the ℓ-th cell and the M base station antennas in the j-th cell,

$$[G_{j\ell}]_{mk} = g_{nmjkl},\ m = 1, \cdots, M,\ k = 1, \cdots, K, \tag{6}$$

V_j is a $M \times K$ matrix of receiver noise whose components are zero-mean, mutually uncorrelated, and uncorrelated with the propagation matrices, and ρ_{p} is a measure of pilot signal-to-noise ratio. We need not quantify ρ_{p} because, as M grows without limit, the effects of the noise vanish.

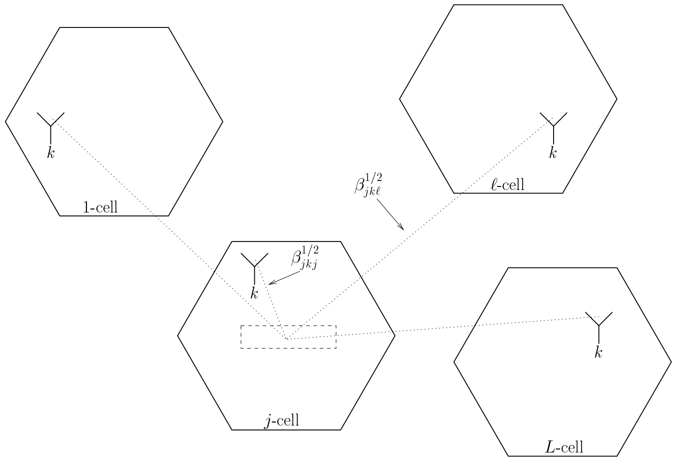

Fig. 2. Reverse-link interference due to pilot contamination for an unlimited number of base station antennas: transmissions from terminals in other cells who use the same pilot sequence interfere with the transmission from the k-th terminal in the j-th cell to his own base station.

IV. Reverse-Link Data Transmission

The K terminals in each cell independently transmit data streams to their respective base station. The base station uses its channel estimate to perform maximum-ratio combining. Fig. 2 illustrates the residual interference to the transmission from the k-th terminal in the j-th cell to its own base station as the number of antennas becomes infinite.

A. Signal model

The j-th base station receives, within each sub-carrier, and within each OFDM symbol, a $M \times 1$ vector comprising transmissions from all of the terminals in the L cells. Again we suppress the dependence on the sub-carrier index,

$$\bar{x}_j = \sqrt{\rho_{\mathrm{r}}} \sum_{\ell=1}^{L} G_{j\ell} \bar{a}_\ell + \bar{w}_j, \tag{7}$$

where $\bar{a}_\ell$ is the $K \times 1$ vector of message-bearing symbols from the terminals of the ℓ-th cell, $\bar{w}_j$ is a vector of receiver noise whose components are zero-mean, mutually uncorrelated, and uncorrelated with the propagation matrices, and ρ_{r} is a measure of signal-to-noise ratio. In our subsequent analysis we assume that the message-bearing signals which are transmitted by the terminals are independent and distributed as zero-mean, unit-variance, complex Gaussian.

B. Maximum-ratio combining

The base station processes its received signal by multiplying it by the the conjugate-transpose of the channel estimate which, according to (5) and (7), yields

$$\begin{aligned} \bar{y}_j &\equiv \hat{G}_{jj}^{\dagger} \bar{x}_j \\ &= \left[\sqrt{\rho_{\mathrm{p}}} \sum_{\ell_1=1}^{L} G_{j\ell_1} + V_j \right]^{\dagger} \left[\sqrt{\rho_{\mathrm{r}}} \sum_{\ell_2=1}^{L} G_{j\ell_2} \bar{a}_{\ell_2} + \bar{w}_j \right], \end{aligned} \tag{8}$$

where the superscript "†" denotes "conjugate transpose". The components of $\bar{y}_j$ comprise sums of inner products between M-component random vectors. As M grows without limit the L2-norms of these vectors grow proportional to M, while the inner products of uncorrelated vectors, by assumption, grow at a lesser rate. For large M, only the products of identical quantities remain significant, i.e., the propagation matrices which appear in both of the bracketed expressions. According to (2) and (6),

$$\frac{1}{M} G_{j\ell_1}^{\dagger} G_{j\ell_2} = D_{\bar{\beta}_{j\ell_1}}^{1/2} \left(\frac{H_{j\ell_1}^{\dagger} H_{j\ell_2}}{M} \right) D_{\bar{\beta}_{j\ell_2}}^{1/2}, \tag{9}$$

where $H_{j\ell}$ is the $M \times K$ matrix of fast fading coefficients between the K terminals of the ℓ-th cell, and the M antennas of the j-th base station, $[H_{j\ell}]_{mk} = h_{nmjk\ell}$, and $D_{\bar{\beta}_{j\ell}}$ is a $K \times K$ diagonal matrix whose diagonal elements comprise the vector $\left[\bar{\beta}_{j\ell}\right]_k = \beta_{jk\ell},\ k = 1, \cdots, K$. As M grows without bound we have

$$\frac{1}{M} H_{j\ell_1}^{\dagger} H_{j\ell_2} \to I_K \delta_{\ell_1 \ell_2}, \tag{10}$$

where I_K is the $K \times K$ identity matrix. The substitution of (10) and (9) into (8) yields

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{r}}}} \bar{y}_j \to \sum_{\ell=1}^{L} D_{\bar{\beta}_{j\ell}} \bar{a}_\ell,\ j = 1, \cdots, L. \tag{11}$$

The k-th component of the processed signal becomes

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{r}}}} y_{kj} \to \beta_{jkj} a_{kj} + \sum_{\ell \neq j} \beta_{jk\ell} a_{k\ell}. \tag{12}$$

The salutary effect of using an unlimited number of base station antennas is that the effects of uncorrelated receiver noise and fast fading are eliminated completely, and transmissions from terminals within one's own cell do not interfere. However transmission from terminals in other cells that use the same pilot sequence constitute a residual interference. The effective signal-to-interference ratio (SIR), which is identical for all sub-carriers but which depends on the indices of the cell and the terminal, is

$$\mathrm{SIR}_{\mathrm{r}k} = \frac{\beta_{jkj}^2}{\sum_{\ell \neq j} \beta_{jk\ell}^2}. \tag{13}$$

The effective signal-to-interference ratio is a random quantity which depends on the random positions of the terminals and the shadow fading coefficients.

Note that the SIR expression (13) is independent of the quantities ρ_{p} and ρ_{r}, and therefore it is independent of the transmitted powers. This is intuitively reasonable: we are operating in a regime where performance is limited only by inter-cell interference, so if every terminal reduces its power by the same factor then the limit SIR is unchanged. Hence we conclude that for an arbitrarily small transmitted energy-per-bit, the SIR (13) can be approached arbitrarily closely by employing a sufficient number of antennas.

A curious thing about the effective SIR is its dependence on the *squares* of the β's. This occurs because the system is operating in a purely interference-limited rather than a noise-limited regime and because of the particular processing which is employed. Prior to maximum ratio combining, the desired signal and the inter-celluar interference are both proportional to the square-roots of their respective β's, while the receiver noise has unit-variance. After maximum-ratio combining, the desired signal and the interference are both proportional to their respective β's, while the noise has standard deviation proportional to the sum of the square-roots of the β's. If the noise were the dominant impairment then the SNR would be the ratio of the β^2 of the desired signal to β of the desired signal, or $\mathrm{SNR} \propto \beta$. But interference is the dominant impairment, so the SIR is proportional to a ratio of squares of β's.

The SIR (13) is constant with respect to frequency because the slow-fading coefficients are independent of frequency. The SIR is constant with respect to the absolute size of the cell, for the following reason. Each of the β-terms is inversely proportional to a range that is raised to the decay exponent, $\beta \propto 1/r^\gamma$. The replacement of the range by the nondimensional quantity, $r \to r/r_c$, does not alter the value of the SIR because the terms r_c^γ appear in both the numerator and denominator and therefore cancel. Consequently the throughput per terminal and the number of terminals which the base station can handle is independent of the cell-size.

C. Reverse-link capacity

Subject to our assumption that the terminals transmit Gaussian message-bearing symbols, the instantaneous capacity of the terminal within each subcarrier is equal to the logarithm of one plus the signal-to-interference ratio. The net throughput per terminal, in units of bits/sec/terminal, accounts for the total bandwidth and frequency re-use, the pilot overhead (the ratio of the time spent sending data to the total slot-length), and the overhead of the cyclic prefix:

$$C_{\mathrm{r}k} = \left(\frac{B}{\alpha}\right)\left(\frac{T_{\mathrm{slot}} - T_{\mathrm{pilot}}}{T_{\mathrm{slot}}}\right)\left(\frac{T_{\mathrm{u}}}{T_{\mathrm{s}}}\right)\log_2\left(1 + \mathrm{SIR}_{\mathrm{r}k}\right), \tag{14}$$

where B is the total bandwidth in Hz, α is the frequency re-use factor (equal to either one, three, or seven in our subsequent analysis), T_{slot} is the slot length, T_{pilot} is the time spent transmitting reverse-link pilots, T_{u} is the useful symbol duration, and T_{s} is the OFDM symbol interval, where the times are measured in seconds.

The net sum throughput per cell, measured in bits/sec/cell is equal to the sum of the net throughputs per terminal,

$$C_{\mathrm{rsum}} = \sum_{k=1}^{K} C_{\mathrm{r}k}. \tag{15}$$

Since the number of terminals that can be served is proportional to the time spent sending pilots, while the instantaneous sum-throughput is proportional to the number of terminals served, it follows that net sum-throughput is maximized by spending approximately half of the slot on sending pilots, and half sending data [8].

V. Forward-Link Data Transmission

Each base station transmits a vector of message-bearing symbols through a pre-coding matrix which is proportional to the conjugate-transpose of its estimate for the forward propagation matrix. As shown in Fig. 3 the transmission from the base station in the ℓ-th cell to its k-th terminal suffers interference from transmissions from the base stations in other cells to their own k-th terminals.

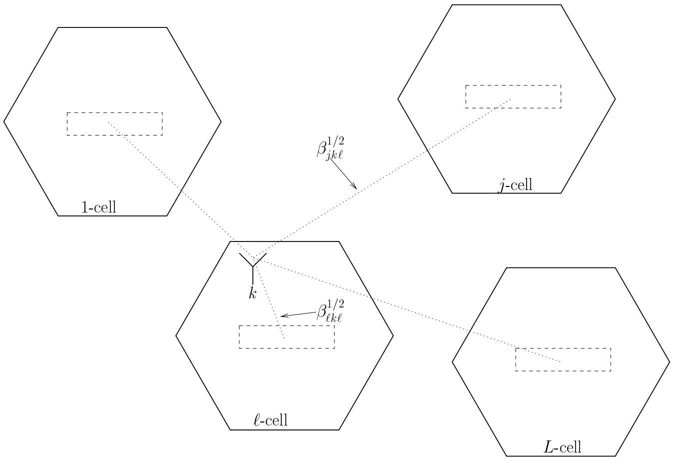

Fig. 3. Forward-link interference due to pilot contamination for an unlimited number of base station antennas: transmissions from base stations in other cells intended for their own k-th terminal interfere with the transmission from the base station in the ℓ-th cell to his k-th terminal.

A. Pre-coding matrix

The j-th base station transmits a $M \times 1$ vector, $\hat{G}_{jj}^{*}\bar{a}_j$, where the superscript "$*$" denotes "complex conjugate", and $\bar{a}_j$ is the vector of message-bearing signals which is intended for the K terminals of the j-th cell. In practice a normalizing factor would be included in order to conform to power constraints. We merely assume that this normalizing factor is the same for all base stations. As M grows without limit the exact value of the normalizing factor is unimportant.

B. Signal model

The K terminals in the ℓ-th cell receive their respective components of a $K \times 1$ vector comprising transmissions from all L base stations,

$$\bar{x}_\ell = \sqrt{\rho_{\mathrm{f}}}\sum_{j=1}^{L} G_{j\ell}^{T}\hat{G}_{jj}^{*}\bar{a}_j + \bar{w}_\ell, \tag{16}$$

$$= \sqrt{\rho_{\mathrm{f}}}\sum_{j=1}^{L} G_{j\ell}^{T}\left[\sqrt{\rho_{\mathrm{p}}}\sum_{\acute{\ell}=1}^{L} G_{j\acute{\ell}} + V_j\right]^{*}\bar{a}_j + \bar{w}_\ell \tag{17}$$

where $\bar{w}_\ell$ is uncorrelated noise, ρ_{f} is a measure of the forward signal-to-noise ratio, the superscript "T" denotes "unconjugated transpose", and we have utilized (5).

We now let the number of base station antennas increase without limit, and again we utilize (9) and (10) to conclude that

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{f}}}}\bar{x}_\ell \to \sum_{j=1}^{L} D_{\bar{\beta}_{j\ell}}\bar{a}_j. \tag{18}$$

The k-th terminal in the ℓ-th cell receives the following:

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{f}}}}x_{k\ell} \to \beta_{\ell k\ell}a_{k\ell} + \sum_{j\neq\ell}\beta_{jk\ell}a_{kj}. \tag{19}$$

The effective signal-to-interference ratio is

$$\mathrm{SIR}_{\mathrm{f}k} = \frac{\beta_{\ell k\ell}^2}{\sum_{j\neq\ell}\beta_{jk\ell}^2}. \tag{20}$$

While the forward and the reverse SIRs, (20) and (13), are described by similar-looking expressions, they in fact have somewhat different statistical characteristics. The numerators have identical statistics. The denominator for the reverse-link SIR (13) is a sum of squares of $L-1$ slow fading coefficients from different terminals to the same base station. These coefficients are statistically independent. The denominator for the forward-link SIR (20) is a sum of squares of $L-1$ slow fading coefficients from different base stations to the same terminal. These coefficients are correlated because motion of the one terminal affects all of the geometric decay factors. Duality as described in [14] does not appear to hold.

C. Forward-link capacity

As in IV-C we translate the forward SIR into the net capacity per terminal (bits/sec/terminal):

$$C_{\mathrm{f}k} = \left(\frac{B}{\alpha}\right)\left(\frac{T_{\mathrm{slot}} - T_{\mathrm{pilot}}}{T_{\mathrm{slot}}}\right)\left(\frac{T_{\mathrm{u}}}{T_{\mathrm{s}}}\right)\log_2\left(1 + \mathrm{SIR}_{\mathrm{f}k}\right), \tag{21}$$

and the net capacity per cell (bits/sec/cell):

$$C_{\mathrm{fsum}} = \sum_{k=1}^{K} C_{\mathrm{f}k}. \tag{22}$$

VI. Numerical Results

Capacity, as derived in IV-C and V-C, is a random quantity, whose randomness is entirely due to terminal positions and shadow fading. In this section we evaluate the cumulative distribution and mean of capacity for a particular scenario.

A. Scenario for numerical study

We assume OFDM parameters identical to LTE (Long-Term Evolution) forward-link parameters: a symbol interval of $T_s = 500/7 \approx 71.4$ microseconds, a subcarrier spacing of $\Delta_f = 15$ kHz, a useful symbol duration $T_u = 1/\Delta_f \approx 66.7$ microseconds, and a guard interval $T_g = T_s - T_u \approx 4.76$ microseconds. The frequency smoothness interval is exactly $N_{\mathrm{smooth}} = 14$ subcarriers. We assume a coherence time of 500 microseconds (equivalent to seven OFDM symbols), of which three symbols are spent sending reverse pilots, and three symbols are spent sending data, either reverse or forward. The remaining one symbol is considered to be additional overhead: in the case of forward data transmission the terminals cannot be expected to process their received pilot signals instantaneously. In the case of reverse data transmission we still count this extra symbol as overhead in order to facilitate comparisons between forward- and reverse-link performance. Within our capacity expressions, (14), (15), (21), and (22), we therefore have a numerical value for the training efficiency term of $(T_{\mathrm{slot}} - T_{\mathrm{pilot}})/T_{\mathrm{slot}} = 3/7$. We serve the maximum possible number of terminals, $K = \tau \cdot N_{\mathrm{smooth}} = 3 \times 14 = 42$.

The spectral efficiency, measured in bits/second/Hz, is independent of bandwidth. For convenient interpretation we assume a total system bandwidth of $B = 20$ MHz. The frequency re-use factor is variously $\alpha = 1, 3, 7$, so the actual bandwith that any cell uses is equal to B/α. The decay exponent is $\gamma = 3.8$, and the shadow-fading standard deviation is $\sigma_{\mathrm{shadow}} = 8.0$ dB. The cell radius is $r_{\mathrm{c}} = 1600$ meters, and the cell-hole radius is $r_{\mathrm{h}} = 100$ meters (though as shown earlier only the ratio $r_{\mathrm{c}}/r_{\mathrm{h}}$ matters). The absolute powers of the base stations and the terminals do not figure in this scenario.

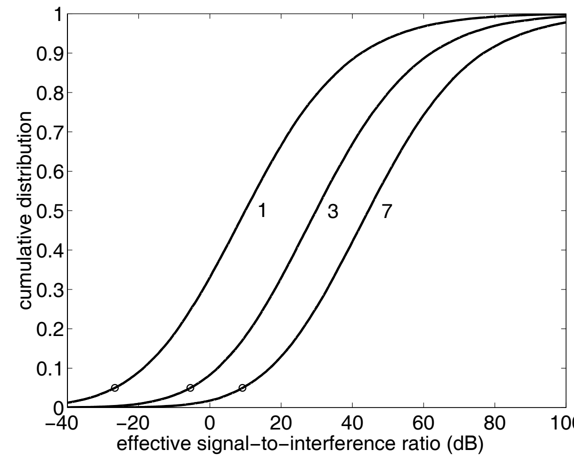

Fig. 4. Cumulative distribution of the reverse effective signal-to-interference ratio (dB) for frequency-reuse factors of one, three, and seven. The circles indicate the $\geq .95$-probability SIRs.

Our simulation comprises the evaluation of the signal-to-interference ratios (13) and (20) for 10^5 independent trials, which translate directly into distributions for SIRs and capacities. We determine the set of cells that interfere with a particular cell by finding all cells which a) reuse the same frequency band, and b) are within eight cell-diameters of that cell.

We note that increasing the coherence interval would not significantly affect the per-terminal capacity, however it would increase proportionately the number of terminals which could be served simultaneously.

B. Reverse-link performance

Fig. 4 shows the cumulative distribution for the reverse effective signal-to-interference ratio (13) for frequency-reuse factors of one, three, and seven. The circles indicate the five-percent values, i.e., the SIR is greater than or equal to the indicated value with probability .95. Frequency re-use of three instead of one increases the SIR by about 21 dB, and re-use of seven adds an additional 15 dB.

Fig. 5 shows the cumulative distribution for the net reverse capacity per terminal (14) for reuse factors of one, three, and seven. Larger reuse factors are beneficial when the SIR is low, the logarithm is in its linear region, and capacity gains due to the large increase in SIR more than offset the loss due to less aggressive frequency reuse which is associated with a reduction in the actual bandwidth that each cell utilizes. When the SIR is already high, a greater frequency reuse factor causes a net decrease in throughput. If the minimum guaranteed performance per terminal is a more important consideration than the mean throughput then a frequency reuse factor of seven should be used.

Table I summarizes the reverse-link performance with respect to the .95-likely SIR, the .95-likely per-terminal net

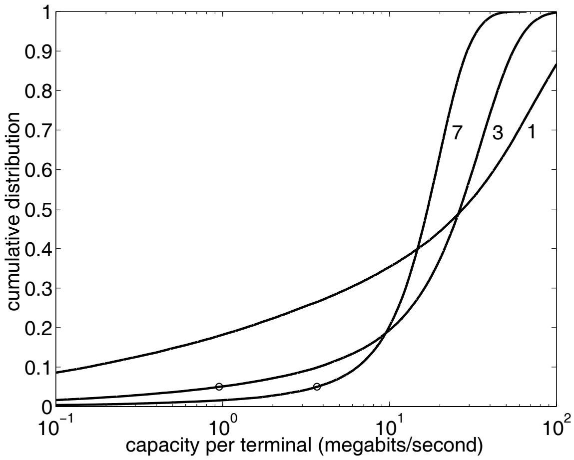

Fig. 5. Cumulative distribution of the net reverse capacity per terminal (megabits/second) for frequency-reuse factors of one, three, and seven. The circles indicate the $\geq .95$-probability capacities.

TABLE I
PERFORMANCE OF REVERSE-LINK FOR FREQUENCY-REUSE FACTORS OF ONE, THREE, AND SEVEN. CAPACITIES ARE EXPRESSED IN MEGABITS/SECOND.

Freq. Reuse	$P \geq .95$ SIR (dB)	$P \geq .95$ Capacity Per Terminal (Mbits/s)	C_{mean} Per Terminal (Mbits/s)	C_{mean} Per Cell (Mbits/s)
1	-27.	.024	44	1800
3	-5.5	.95	28	1200
7	9.1	3.6	17	730

capacity, the mean net capacity per terminal, and the mean net capacity per cell for frequency-reuses of one, three, and seven.

C. Forward-link performance

The forward-link performance is similar to the reverse-link performance, although the statistics of the SIRs are somewhat different.

Fig. 6 shows the cumulative distribution for the forward effective signal-to-interference ratio (20) for frequency-reuse factors of one, three, and seven.

Fig. 7 shows the cumulative distribution for the net forward capacity per terminal (21) for reuse factors of one, three, and seven.

Table II summarizes the forward-link performance with respect to .95-likely effective SIR, the .95-likely per-terminal net capacity, the mean net capacity per terminal, and the mean net capacity per cell for frequency-reuse of one, three, and seven.

VII. COMMENTS

A. How many antennas is "unlimited"?

The assumption of an unlimited number of antennas at the base station greatly simplifies the analysis, and it illustrates the desirable effects of operating with a large excess of antennas compared with terminals. It is reasonable to ask what is the optimum number of antennas from a cost-effectiveness point of view. We do not attempt to answer that question

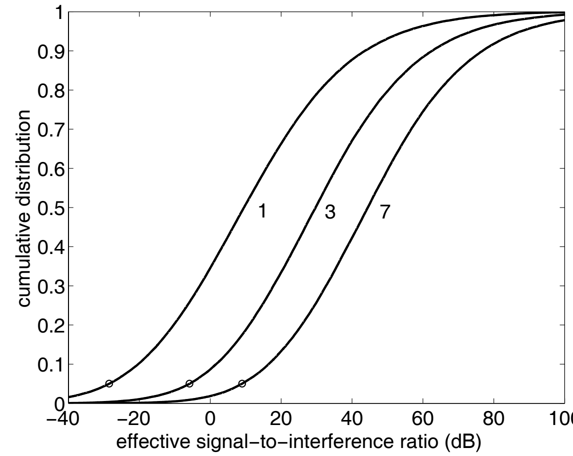

Fig. 6. Cumulative distribution of the forward effective signal-to-interference ratio (dB) for frequency-reuse factors of one, three, and seven. The circles indicate the $\geq .95$-probability SIRs.

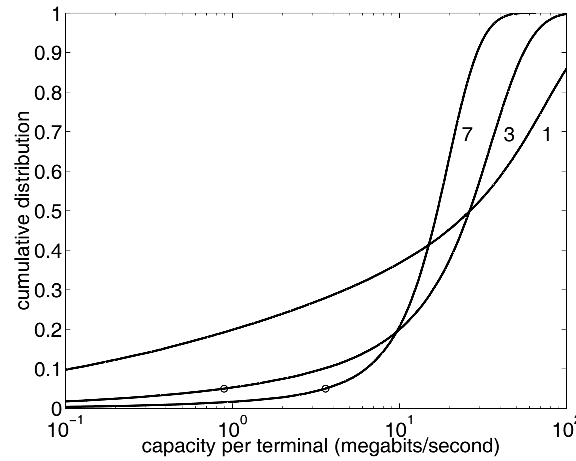

Fig. 7. Cumulative distribution of the net forward capacity per terminal (megabits/second) for frequency-reuse factors of one, three, and seven. The circles indicate the $\geq .95$-probability capacities.

here, but a definitive answer must depend on the details of the propagation, the complexity of the signal processing, and the cost of antenna elements. It is hoped that considerable economy would be realized in manufacturing large numbers of low-power transmit/receive units in place of small numbers of high-power units.

B. Propagation assumptions

Our analysis assumes that inner products between propagation vectors of different terminals grow at a lesser rate than inner products of propagation vectors with themselves. Clearly experimental work is needed to discover the range of validity of this assumption.

C. More complicated pre-coding and combining

We analyzed the simplest linear pre-coding and combining. One could consider more complicated operations, for example replacing the conjugate transpose of the channel estimate by its pseudo-inverse, or possibly using dirty-paper coding on the forward link and sphere-decoding on the reverse link.

TABLE II
PERFORMANCE OF FORWARD-LINK FOR FREQUENCY-REUSE FACTORS OF ONE, THREE, AND SEVEN. CAPACITIES ARE EXPRESSED IN MEGABITS/SECOND.

Freq. Reuse	$P \geq .95$ SIR (dB)	$P \geq .95$ Capacity Per Terminal (Mbits/s)	C_{mean} Per Terminal (Mbits/s)	C_{mean} Per Cell (Mbits/s)
1	-29.	.016	44	1800
3	-5.8	.89	28	1200
7	8.9	3.6	17	730

D. Variable-length slots for variable mobility

The scenario that we focused on assumes a temporal coherence of 500 microseconds. If we associate the coherence time with the time that it takes a terminal to move no more than 1/4 wavelength, then for a carrier frequency of 1.9 GHz, the 500 microsecond slot could accommodate any terminal moving slower than 80 meters/second (180 miles/hour). It would be advantageous to group slower terminals together for simultaneous service in longer coherence intervals. Thus a 1000 microsecond coherence slot would permit the simultaneous service of 84 terminals. The longer coherence interval would not change the per-terminal throughput, but the per-cell throughput would be doubled. A variable-length slot structure would, however, require a more complicated control layer.

E. MIMO terminals

Multiple antennas at the terminals would increase the throughput per terminal proportionately. However the amount of pilot resources per terminal would also increase at the same rate. Consequently the number of terminals that could be served simultaneously would be reduced by a commensurate amount, so the average throughput per cell would remain the same.

F. Effects of using different pilots in different cells, or serving fewer terminals

Our analysis assumes that exactly the same set of pilot sequences is used in all active cells, and that each cell serves the maximum possible number of terminals. Here we assume that different cells use different sets of orthogonal pilot sequences, and we allow a reduction in the number of terminals. Recall that reverse-link pilots are transmitted over a $\tau \times N_{\mathrm{smooth}}$ time-frequency space. Within a cell each terminal is assigned a τN_{smooth} pilot sequence which is orthogonal to the pilot sequences that are assigned to the other terminals in the same cell. Collectively the $K \leq \tau N_{\mathrm{smooth}}$ terminals in the ℓ-th cell have the set of pilot sequences represented by Φ_ℓ - a $\tau N_{\mathrm{smooth}} \times K$ unitary matrix such that $\Phi_\ell^\dagger \Phi_\ell = I_K$. In general pilots from different cells are not orthogonal, unless, of course, $K \cdot L \leq \tau N_{\mathrm{smooth}}$. Each base station correlates its received pilot signals with its own orthogonal pilot signals. All terminals in the other cells contribute to pilot contamination. The j-th base station obtains the following channel estimate:

$$\hat{G}_{jj} = \sqrt{\rho_{\mathrm{p}}} G_{jj} + \sqrt{\rho_{\mathrm{p}}} \sum_{\ell \neq j} G_{j\ell} \Phi_\ell^T \Phi_j^* + V_j. \tag{23}$$

The analysis is straightforward, and we merely state the results.

1) Reverse link: The reverse-link data signal, after maximum-ratio combining (formerly (11)) becomes

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{r}}}} \bar{y}_j \to D_{\bar{\beta}_{jj}} \bar{a}_j + \sum_{\ell \neq j} \Phi_j^T \Phi_\ell^* D_{\bar{\beta}_{j\ell}} \bar{a}_\ell, \; j = 1, \cdots, L, \tag{24}$$

which yields an effective SIR for the received transmission of the k-th terminal in the j-th cell of

$$\mathrm{SIR_r} = \frac{\beta_{jkj}^2}{\sum_{\ell \neq j} \bar{\phi}_{kj}^T \Phi_\ell^* D_{\bar{\beta}_{j\ell}}^2 \Phi_\ell^T \bar{\phi}_{kj}^*}, \tag{25}$$

where $\bar{\phi}_{kj}$ is the k-th column-vector of Φ_j. We now assume that the unitary pilot-sequence matrices, $\{\Phi_\ell\}$ are chosen randomly and independently according to the isotropic (Haar measure) distribution. It can be shown that the vector $\bar{\phi}_{kj}^T \Phi_\ell^*$ has exactly the same probability distribution as does any row vector of Φ_ℓ [15], [16]. In turn, any element of Φ_ℓ has a standard deviation that is equal to $1/\sqrt{\tau N_{\mathrm{smooth}}}$. If we approximate the denominator by its expectation, we have

$$\mathrm{SIR_r} \approx \left(\frac{\tau N_{\mathrm{smooth}}}{K}\right) \frac{\beta_{jkj}^2}{\sum_{\ell \neq j} \frac{1}{K} \sum_{\acute{k}=1}^{K} \beta_{j\acute{k}\ell}^2}. \tag{26}$$

Thus the effect of using different random orthogonal pilot sequences in each cell is to average the interference over all of the terminals in the interfering cells, and we don't expect any major difference in the typical SIR with respect to reusing the pilots from cell to cell. If one serves fewer than the maximum possible number of terminals then the SIR is increased, which may benefit the .95-likely per-terminal capacity. However the mean throughput per cell will suffer (the reduction of K outside the logarithm in (15) will more than compensate for the increase in the SIR which occurs inside of the logarithm).

2) Forward link: The forward-link data signal (formerly (18)) becomes

$$\frac{1}{M\sqrt{\rho_{\mathrm{p}}\rho_{\mathrm{r}}}} \bar{x}_\ell \to D_{\bar{\beta}_{\ell\ell}} \bar{a}_\ell + \sum_{j \neq \ell} D_{\bar{\beta}_{j\ell}} \Phi_\ell^\dagger \Phi_j \bar{a}_j, \tag{27}$$

which yields the effective SIR for the k-th terminal in the ℓ-th cell of

$$\mathrm{SIR_f} = \frac{\beta_{\ell k\ell}^2}{\sum_{j \neq \ell} \beta_{jk\ell}^2 \bar{\phi}_{k\ell}^\dagger \Phi_j \Phi_j^\dagger \bar{\phi}_{k\ell}}. \tag{28}$$

When the maximum number of terminals is used then $\Phi_j \Phi_j^\dagger = I_K$ and the above SIR expression reduces exactly to the SIR that results from reusing the same pilot sequences in different cells. Again, if independent sets of random orthogonal pilot sequences are used, and if we approximate the denominator by its expectation, then

$$\mathrm{SIR_f} \approx \left(\frac{\tau N_{\mathrm{smooth}}}{K}\right) \frac{\beta_{\ell k\ell}^2}{\sum_{j \neq \ell} \beta_{jk\ell}^2}, \tag{29}$$

and our conclusions are similar to our conclusions for the reverse link.

To summarize: using different pilots in different cells makes little difference. Reducing the number of terminals in service will increase their typical SIR's, but overall it will reduce throughput. Less aggressive frequency re-use is a more effective measure, as it targets the typically most troublesome sources of interference.

G. Effects of asynchronous operation

Our analysis assumes that receptions from the different cells are synchronized. A little reflection makes it clear that this is the worst possible situation precisely because it maximizes the pilot contamination. For example, if during the time when the j-th cell's terminals were transmitting their pilots, the base stations in all other cells were transmitting data, then these inter-cellular transmissions would constitute uncorrelated noise, and in the limit of an infinite number of antennas, the j-th cell would experience no inter-cellular interference.

H. Effects of cell-size

We noted earlier in IV-B that the number of terminals that the base station can handle and the throughput per terminal are independent of the absolute size of the cell. Consequently an ever-increasing user-density (e.g., terminals per unit-area) can be served by utilizing an increasing number of smaller cells.

I. Inter-Cellular Cooperation

The scheme that we analyzed entails no cooperation between cells. Here we mention two types of possible inter-cellular cooperation.

1) Selectively assigning terminals to cells: In our analysis if a terminal is geographically inside of a cell, then it is assigned to that base station. Better performance would result if the terminal were assigned to the base station to which it enjoys the strongest channel.

2) Cooperative MIMO operation: Cooperative MIMO operation (also called "network MIMO") has been proposed in which a multiplicity of base stations are, in effect, wired together to create a distributed antenna array which performs multi-user MIMO activities [17], [18], [19], [20]. We merely point out that in the above references there is no treatment of the burden of acquiring CSI, rather this information apparently is assumed to be available for free. Moreover the results do not show any dependence on the mobility of the terminals. Pilot contamination must also figure in a cooperative MIMO system if pilot sequences are reused by other clusters of base stations. A cooperative MIMO system would require extensive backhaul links among the cells.

J. FDD (Frequency Division Duplex) operation

It is reasonable to ask whether the proposed system could be implemented as a FDD system. One approach would use forward-link pilots to inform the terminals of the forward channel, and the CSI would be transmitted to the base station on the reverse link. However the required duration of the training interval is now proportional to the number of base station antennas. If, again, three OFDM symbols were used for forward pilots, then at most 42 base station antennas could be accommodated. To preserve a large excess of antennas over terminals would force a drastic reduction in the number of terminals served. There would also be additional overhead for transmitting the CSI on the reverse link.

An alternative FDD scheme would depend on the correctness of the untested conjecture that the forward- and reverse-channels of an FDD system, although statistically independent, have similar spatial eigenvalues when corrected for wavelength differences [21], [22]. The scheme would derive information about the forward link entirely from reverse-link transmissions. Its success would rest on the growth of the number of significant eigenvalues with the number of antennas: enough to handle the K terminals, but not too many. Again, new propagation experiments are needed.

VIII. Conclusions

The acquisition of channel state information and the phenomenon of pilot contamination impose fundamental limitations on what can be achieved with a noncooperative cellular multiuser MIMO system. Notwithstanding these limitations, we have outlined a compelling case for a time-division duplex cellular system which employs base stations equipped with large numbers of antennas that communicate simultaneously with smaller numbers of cheap, single-antenna terminals through multi-user MIMO techniques. This system has the potential to deliver high throughputs reliably on both the forward and the reverse link in fast-changing propagation environments. As the number of base station antennas grows without limit all of the effects of uncorrelated noise and fast fading disappear. What remains is inter-cellular interference that results from pilot contamination.

The use of a large excess of base station antennas compared with the number of terminals which are being served permits the simplest sort of precoding on the forward link and processing on the reverse link. In the limit of an infinite number of antennas a multicellular analysis, which explicitly accounts for the pilot overhead and channel estimation error, is exceedingly simple.

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