Communication modes with partially coherent fields

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We develop a theory for the description of partially coherent wave fields in linear optical systems in terms of the so-called communication modes. The communication modes are the singular functions and singular values of the appropriate propagation kernels. In particular, we show that optical fields of any state of coherence may be readily propagated through deterministic systems using the modal representation based on the system properties. The relation of the communication modes to the conventional coherent-mode representation is discussed, and expressions for the effective degree of coherence in the optical system are derived. The results are illustrated by numerical examples in optical near-field geometry. © 2007 Optical Society of America

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1. INTRODUCTION

Real-world optical fields are never fully coherent, and the wide use of new types of light sources, such as LEDs or semiconductor lasers, has increased the practical need for methods of modeling partially coherent light in a variety of situations. Optical coherence theory provides a well-established basis for that purpose [1], but often the means of getting some specific knowledge of the behavior of fields in optical systems becomes mathematically complicated or numerically demanding. One option for facilitating the analysis is to use the coherent-mode expansion, in which a partially coherent field is expressed as an incoherent sum of fully coherent contributions [1–4]. In this paper we present an alternative modal approach for the analysis of partially coherent fields in optical systems, based on the so-called communication modes.

The communication modes [5] have proved to be a useful concept for the study of resolution, propagation, field synthesis, and information content of coherent optical waves [6–9], even in the context of optical near fields [10]. They have also been used for the evaluation of the performance limits of linear optical components [11]. The coherent communication modes are the singular functions of the propagation operator, which operates on the field over some region of space and gives as a result the field over some other region of space (see, for instance, [12] and the original studies cited therein). Some of the attractive properties of the communication modes are that in normal situations they form complete orthonormal sets in the source and observation domains and that there is a one-to-one coupling between the source and receiving modes. Moreover, the communication modes are invariant in the sense that they are defined solely by the properties of the optical system, not in terms of the field.

To extend the applicability of the communication-mode method, we develop in this paper a corresponding representation for partially coherent light. The statistical properties of partially coherent wave fields can be propagated in linear optical systems in a manner that is analogous to the propagation of the field itself, and we derive formally the partially coherent communication modes as the singular functions of such coherence propagation. In deterministic media and systems it is found that these modes are spatially fully coherent and can be expressed as products of the coherent communication modes.

Some concepts based on a similar approach have recently been discussed in connection with applications in millimeter-wave interferometry [13,14]. Here we formulate a general theory that can be applied to the study of stationary, partially coherent optical fields in free-space propagation as well as in many kinds of linear systems. In Section 2 we briefly recall the basics of the communication modes representation of coherent fields, and in Section 3 the same notions are generalized for spectrally partially coherent scalar waves. In particular, we demonstrate how the communication modes of partially coherent light can be solved in deterministic media and optical systems. The propagation of partially coherent fields in terms of the communication modes is then discussed in Section 4. In Section 5 we also compare this approach to the method that relies on the traditional coherent-mode expansion. In addition, in Section 6 we find expressions of the effective degree of coherence of wave fields on the basis of the communication-mode expansion coefficients. The results are illustrated in Section 7 by numerical examples related to partially coherent beams in an optical near-field geometry. The main conclusions are summarized in Section 8.

2. COMMUNICATION MODES FOR COHERENT FIELDS

As a basis for the development of the communication-mode representation for partially coherent fields, we first briefly introduce the notation and review the foundations of the communication modes of coherent fields. Let us assume that a coherent field U_0 occupies a region A, which, for instance, may be an aperture in a planar screen. When propagated through a deterministic optical system, e.g., simply some distance in free space, the resulting field U in a region O can be expressed as

$$U(\mathbf{r}) = \int_A G(\mathbf{r},\boldsymbol{\rho})U_0(\boldsymbol{\rho})\mathrm{d}^2\rho, \tag{1}$$

where G is the Green function of the system. For physically realizable systems we may assume that the integral operation in Eq. (1) is of the Hilbert–Schmidt class, which formally means that $|G|^2$ integrated over both domains A and O remains finite [15]. This is the case, for instance, when G is bounded and A and O are finite. The Green function can then be expanded as [15,16]

$$G(\mathbf{r},\boldsymbol{\rho}) = \sum_{n=0}^{\infty} g_n\phi_n(\mathbf{r})\psi_n^*(\boldsymbol{\rho}), \tag{2}$$

where g_n, ψ_n, and ϕ_n are the solutions to the eigenequations

$$|g_n|^2\psi_n(\boldsymbol{\rho}) = \int_A K_a(\boldsymbol{\rho},\boldsymbol{\rho}')\psi_n(\boldsymbol{\rho}')\mathrm{d}^2\rho', \tag{3}$$

$$|g_n|^2\phi_n(\mathbf{r}) = \int_O K_o(\mathbf{r},\mathbf{r}')\phi_n(\mathbf{r}')\mathrm{d}^2r', \tag{4}$$

in which the kernels are defined as

$$K_a(\boldsymbol{\rho},\boldsymbol{\rho}') = \int_O G^*(\mathbf{r},\boldsymbol{\rho})G(\mathbf{r},\boldsymbol{\rho}')\mathrm{d}^2r, \tag{5}$$

$$K_o(\mathbf{r},\mathbf{r}') = \int_A G(\mathbf{r},\boldsymbol{\rho})G^*(\mathbf{r}',\boldsymbol{\rho})\mathrm{d}^2\rho. \tag{6}$$

The functions ψ_n and ϕ_n are the so-called communication modes of the optical system, and they form, when normalized, complete orthonormal sets in their respective domains. It follows from Eq. (2) that the mode functions are connected by

$$\int_A G(\mathbf{r},\boldsymbol{\rho})\psi_n(\boldsymbol{\rho})\mathrm{d}^2\rho = g_n\phi_n(\mathbf{r}), \tag{7}$$

$$\int_O G^*(\mathbf{r},\boldsymbol{\rho})\phi_n(\mathbf{r})\mathrm{d}^2r = g_n^*\psi_n(\boldsymbol{\rho}), \tag{8}$$

where the orthonormality of the modes has been used.

In the traditional cases of Fraunhofer and Fresnel diffraction, closed-form solutions to the eigenequations exist. For Fraunhofer diffraction with rectangular apertures, the communication modes are prolate spheroidal wave functions (PSWFs) [17], and in the Fresnel domain they are closely related to the PSWFs [6,18]. From Eqs. (7) and (8) it is evident that there is a one-to-one coupling between the source and receiving communication modes. The coefficients g_n define the coupling strengths between each pair of mode functions. The coefficients are usually ordered so that $|g_0| \geq |g_1| \geq \cdots \geq |g_n| \geq |g_{n+1}| \geq \cdots$. After some limit $n=N$ the coupling coefficients become so small that in the presence of noise, i.e., in all realistic situations, the corresponding modes do not effectively contribute to the observed field. In that case, the propagation operator of Eq. (2) can be expressed as a truncated sum of the first $N+1$ modes.

3. COMMUNICATION MODES FOR PARTIALLY COHERENT FIELDS

We now proceed to extend the theory for stationary, partially coherent scalar fields. In the space–frequency domain, the statistical properties of the waves can be expressed in terms of the cross-spectral density [1,2]

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle, \tag{9}$$

where $U(\mathbf{r},\omega)$ is a realization that represents the field at frequency ω and the angle brackets denote ensemble averaging. We thus consider the spatial coherence properties of the wave field at a single frequency, but in the following the frequency dependence is suppressed from the notation. However, we emphasize that all the functions and constants to be introduced are generally frequency dependent.

For arbitrary partially coherent wave fields traversing a linear optical system, we can write, instead of Eq. (1), the expression

$$W(\mathbf{r}_1,\mathbf{r}_2) = \int\int_A Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2, \tag{10}$$

where W_0 and W are the cross-spectral densities of the field in regions A and O, respectively, and Q is the propagation kernel corresponding to the system. It is assumed in Eq. (10) that the randomness of the field and the possible fluctuations of the optical system are uncorrelated. As in the coherent case, if the propagation kernel Q is bounded and if the domains A and O are finite, the coherence propagator can be expanded biorthogonally as

$$Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \sum_n d_n\Phi_n(\mathbf{r}_1,\mathbf{r}_2)\Psi_n^*(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2). \tag{11}$$

Now d_n, Ψ_n, and Φ_n are the solutions to the eigenequations

$$|d_n|^2\Psi_n(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \int\int_A H_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')\Psi_n(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2', \tag{12}$$

$$|d_n|^2\Phi_n(\mathbf{r}_1,\mathbf{r}_2)=\int\int_O H_o(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_1',\mathbf{r}_2',)\Phi_n(\mathbf{r}_1',\mathbf{r}_2')\mathrm{d}^2r_1'\mathrm{d}^2r_2', \tag{13}$$

where the kernels are

$$H_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')=\int\int_O Q^*(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')\times\mathrm{d}^2r_1\mathrm{d}^2r_2, \tag{14}$$

$$H_o(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_1',\mathbf{r}_2')=\int\int_A Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)Q^*(\mathbf{r}_1',\mathbf{r}_2',\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\times\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \tag{15}$$

From Eq. (11) we find the following relationship between the communication modes in the source and receiving domains:

$$\int\int_A Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\Psi_n(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2=d_n\Phi_n(\mathbf{r}_1,\mathbf{r}_2), \tag{16}$$

and similarly for the transformation from the receiving to transmitting modes:

$$\int\int_O Q^*(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\Phi_n(\mathbf{r}_1,\mathbf{r}_2)\mathrm{d}^2r_1\mathrm{d}^2r_2=d_n^*\Psi_n(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2). \tag{17}$$

To obtain these relations we again made use of the orthonormality of the modes.

Generally, the communication modes associated with partially coherent wave fields can be solved from the integral equations above [e.g., Eqs. (12) and (13)] using the appropriate propagation kernel Q in the same way as in the coherent case, but the problem now has more dimensions. However, for deterministic optical systems we obtain the solution relatively easily by employing the communication modes of coherent systems. In particular, in view of Eqs. (1), (9), and (10), the propagation kernel of the cross-spectral density in a deterministic linear system can be expressed rather obviously as

$$Q(\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=G^*(\mathbf{r}_1,\boldsymbol{\rho}_1)G(\mathbf{r}_2,\boldsymbol{\rho}_2). \tag{18}$$

It is clear that Q is bounded whenever G is bounded, i.e., the expansion in Eq. (11) is valid for all practical deterministic systems. If we take ψ_n, ϕ_n, and g_n to be the communication modes and the coupling coefficients for the Green function G in the coherent case, it follows at once that the functions

$$\Psi_n(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\Psi_{ml}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\psi_m^*(\boldsymbol{\rho}_1)\psi_l(\boldsymbol{\rho}_2), \tag{19}$$

$$\Phi_n(\mathbf{r}_1,\mathbf{r}_2)=\Phi_{ml}(\mathbf{r}_1,\mathbf{r}_2)=\phi_m^*(\mathbf{r}_1)\phi_l(\mathbf{r}_2), \tag{20}$$

and the coefficients

$$d_n=d_{ml}=g_m^*g_l \tag{21}$$

are the solutions to Eqs. (12) and (13). Thus, Eqs. (19)–(21) correspond to the communication modes and coupling coefficients for partially coherent fields in any deterministic optical system corresponding to the Green function G. Furthermore, it follows directly from the factored form of Eqs. (19) and (20) that the communication modes in this case are spatially completely coherent [19].

The method described above for solving the communication modes for partially coherent fields is valid for any deterministic optical systems in which the propagator Q is separable, as in Eq. (18). This includes, for example, free-space propagation between two apertures in various geometries [6,8,9,20], diffraction in imaging systems [21–23], and deterministic radiation and scattering phenomena [7,11,24–26]. Most of these studies deal with coherent light only, although some effects of incoherence and partial coherence have been addressed [27–29]. In the following sections we restrict ourselves to deterministic systems, but we focus on partially coherent light. It should be emphasized, however, that the general approach based on Eqs. (11)–(15) can be used for finding the communication modes even in more complex cases, such as propagation in random media and through other fluctuating optical systems.

4. PROPAGATION OF PARTIALLY COHERENT FIELDS

The communication modes, defined for the factorable coherence-propagation kernel in the two domains as described in Section 3, provide two basis sets of orthonormal, completely coherent functions, given by Eqs. (19) and (20). Consequently, the cross-spectral density in the source aperture can be expanded in the communication modes as

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}A_{ml}\psi_m^*(\boldsymbol{\rho}_1)\psi_l(\boldsymbol{\rho}_2), \tag{22}$$

where

$$A_{ml}=\int\int_A \psi_m(\boldsymbol{\rho}_1)\psi_l^*(\boldsymbol{\rho}_2)W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2. \tag{23}$$

It follows directly from Eqs. (10), (16), and (19)–(21) that the cross-spectral density in the observation domain then is

$$W(\mathbf{r}_1,\mathbf{r}_2)=\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}A_{ml}g_m^*g_l\phi_m^*(\mathbf{r}_1)\phi_l(\mathbf{r}_2), \tag{24}$$

i.e., the propagation operation in an optical system is reduced to a simple sum of the modes $\phi_m^*\phi_l$ in the observation domain, multiplied by the source-field projections A_{ml} and coupling coefficients $g_m^*g_l$. The optical intensity distribution (at the frequency in question) across the receiving aperture is obtained from the spectral density, defined as $S(\mathbf{r})=W(\mathbf{r},\mathbf{r})$.

The projection coefficients A_{ml} contain all the information about the state of coherence of the field. Since the cross-spectral density function is Hermitian, $W(\mathbf{r}_2,\mathbf{r}_1)$

$=W^{*}(\mathbf{r}_1,\mathbf{r}_2)$, these coefficients have the general property that $A^{*}_{ml}=A_{lm}$. In the special case of a spatially completely coherent source field, the cross-spectral density in the transmitting aperture can be expressed as $W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=U_0^{*}(\boldsymbol{\rho}_1)U_0(\boldsymbol{\rho}_2)$, and we find, rather obviously,

$$A_{ml}=\int U_0^{*}(\boldsymbol{\rho}_1)\psi_m(\boldsymbol{\rho}_1)\mathrm{d}^2\rho_1\int U_0(\boldsymbol{\rho}_2)\psi_l^{*}(\boldsymbol{\rho}_2)\mathrm{d}^2\rho_2=a_m^{*}a_l, \tag{25}$$

where a_m and a_l are the projections of the coherent field onto the communication modes of the coherent system. Hence, the cross-spectral density in Eq. (24) factors in the two variables, and the field in the receiving domain is also spatially coherent, as expected physically. On the other hand, if we assume that the source field is spatially incoherent, with an intensity distribution $I_0(\boldsymbol{\rho})$ across the transmitting region, the cross-spectral density can be taken to be of the form $W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=I_0(\boldsymbol{\rho}_1)\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$, where $\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$ is the Dirac delta function [30]. In such a case, Eq. (23) yields

$$A_{ml}=\int_A I_0(\boldsymbol{\rho}_1)\psi_m(\boldsymbol{\rho}_1)\psi_l^{*}(\boldsymbol{\rho}_1)\mathrm{d}^2\rho_1. \tag{26}$$

A combination of Eqs. (24) and (26), which is a form of the van Cittert–Zernike theorem [1] for optical systems, shows that the field in the observation domain now is, in general, spatially partially coherent. If the transmitting intensity further is a constant, $I_0(\boldsymbol{\rho})=I_0$, owing to the orthonormality of the modes we simply obtain $A_{ml}=I_0\delta_{ml}$, where δ_{ml} is the Kronecker delta.

5. COMPARISON WITH THE COHERENT-MODE REPRESENTATION

In Section 4 we already mentioned the case of a fully coherent source, which can be considered as a special field that contains only a single coherent mode. In general, the coherent-mode expansion of a partially coherent wave field takes on the form [1,2]

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\sum_n \lambda_n\alpha_n^{*}(\boldsymbol{\rho}_1)\alpha_n(\boldsymbol{\rho}_2), \tag{27}$$

where α_n and λ_n are the eigenfunctions and the eigenvalues of the Fredholm-type integral equation in domain A,

$$\int_A W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\alpha_n(\boldsymbol{\rho}_1)\mathrm{d}^2\rho_1=\lambda_n\alpha_n(\boldsymbol{\rho}_2), \tag{28}$$

with the cross-spectral density W_0 as the kernel. The representation in Eq. (27) resembles the expansion in Eq. (22); however, there are important differences. Conceptually, the most important difference is that the communication-modes, unlike the coherent modes, are the modes of the *system*. The effect of the optical system is contained in the mode functions and coupling coefficients. This could be an advantage compared with the coherent modes in the context of field and coherence propagation.

Mathematically, the expansions in Eqs. (22) and (27) differ in that the communication modes do not diagonalize the function W_0, i.e., the coefficients A_{ml} are nonzero for $m\neq l$, making the communication-mode expansion in that sense a less efficient representation of the cross-spectral density. However, we may readily diagonalize the communication-mode expansion in Eq. (22). Indeed, since the matrix $A=\{A_{ml}\}$ is Hermitian, it can be expressed in the form

$$A=U\Lambda U^{\dagger}, \tag{29}$$

where U is a unitary matrix and Λ is a diagonal matrix with elements λ'_k, $k=0,1,2,\ldots$. The elements of A then are

$$A_{ml}=\sum_k U_{mk}\lambda'_k(U^{\dagger})_{kl}=\sum_k U_{mk}\lambda'_k U^{*}_{lk}, \tag{30}$$

and the cross-spectral density in Eq. (22) becomes

$$\begin{aligned} W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)&=\sum_{ml}\sum_k U_{mk}\lambda'_k U^{*}_{lk}\psi_m^{*}(\boldsymbol{\rho}_1)\psi_l(\boldsymbol{\rho}_2)\\ &=\sum_k \lambda'_k\left[\sum_m U^{*}_{km}\psi_m^{*}(\boldsymbol{\rho}_1)\right]\left[\sum_l U_{kl}\psi_l(\boldsymbol{\rho}_2)\right], \end{aligned} \tag{31}$$

since $U_{mk}=U^{*}_{km}$ due to unitarity. Hence, comparison with the expansion in Eq. (27) shows that the coherent-mode eigenvalues are the diagonal elements λ'_k and the coherent modes are $\alpha_k=\Sigma_l U_{kl}\psi_l$, where ψ_l are the system's communication modes.

Regarding the propagation of the coherence properties of a field, the coherent-mode representation [Eq. (27)] provides a simpler way than a direct solution of the basic integrals of the form of Eq. (10). More specifically, one can propagate the mode functions separately, and the dimensionality of the integrals to be solved is thus reduced. However, for studying the propagation of different partially coherent fields in a specified optical system, the set of coherent modes and their eigenvalues must first be evaluated for each different cross-spectral density. In addition, the number of significant modes that are needed generally increases as the level of coherence of the field is decreased.

If expanded in the communication modes [Eq. (22)], the propagation of the cross-spectral density becomes very simple. Once the modes have been established from the Green function of the system, the propagation consists of merely finding the expansion coefficients of the arbitrary cross-spectral density in the source aperture according to Eq. (23), multiplying these with the coupling coefficients, and carrying out the summation of the receiving modes, as specified in Eq. (24). Furthermore, as discussed at the end of Section 2, the number of the communication modes affecting the field in the observation domain depends on the coupling coefficients (and the level of noise), i.e., the properties of the system, instead of the state of coherence of the field.

6. EFFECTIVE DEGREE OF COHERENCE

Since the communication modes are orthonormal functions, expressions can be derived for the various field properties in terms of the modes' coupling coefficients, which are similar to those previously obtained for the coherent-mode representation [2].

As a preliminary step, let us first consider the field in the source aperture, described by the expansion in Eq. (22). The spectral density is then, by definition,

$$S_0(\boldsymbol{\rho}) = W_0(\boldsymbol{\rho},\boldsymbol{\rho}) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} A_{ml}\psi_m^*(\boldsymbol{\rho})\psi_l(\boldsymbol{\rho}). \tag{32}$$

Integrating this over the aperture, we find the relation

$$\int_A S_0(\boldsymbol{\rho})\mathrm{d}^2\rho = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} A_{ml}\delta_{ml} = \sum_{m=0}^{\infty} A_{mm}, \tag{33}$$

which follows directly from the orthonormality of mode functions ψ_m. This produces the interesting result that only the communication modes of the symmetric form $\psi_m^*\psi_m$ contribute to the integrated spectral density in the aperture. The effect of the other (nondiagonal) terms is canceled out in the integration, but they may still locally affect the intensity distribution. In the same way, we may evaluate the squared absolute value of the cross-spectral density integrated over the aperture. This readily yields

$$\int\int_A |W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2\mathrm{d}^2\rho_1\,\mathrm{d}^2\rho_2 = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty}|A_{ml}|^2, \tag{34}$$

where the orthonormality of the modes was used.

We may now turn our attention to the effective degree of coherence, which in a domain D is defined through the classic formula [31]

$$\mu^2 = \frac{\int\int_D |W(\mathbf{r}_1,\mathbf{r}_2)|^2\mathrm{d}^2r_1\mathrm{d}^2r_2}{\int\int_D S(\mathbf{r}_1)S(\mathbf{r}_2)\mathrm{d}^2r_1\mathrm{d}^2r_2}. \tag{35}$$

We note that on introducing the complex degree of spatial (spectral) coherence [1]

$$\mu(\mathbf{r}_1,\mathbf{r}_2) = \frac{W(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{S(\mathbf{r}_1)S(\mathbf{r}_2)}}, \tag{36}$$

Eq. (35) assumes the form

$$\mu^2 = \frac{\int\int_D S(\mathbf{r}_1)S(\mathbf{r}_2)|\mu(\mathbf{r}_1,\mathbf{r}_2)|^2\mathrm{d}^2r_1\mathrm{d}^2r_2}{\int\int_D S(\mathbf{r}_1)S(\mathbf{r}_2)\mathrm{d}^2r_1\mathrm{d}^2r_2}, \tag{37}$$

showing that μ describes, in effect, an average degree of coherence of the field in D, weighted by the spectral density. The effective degree of coherence has attracted renewed interest, and its mathematical and physical properties have recently been extensively studied in a variety of contexts [4,32–36]. It has also proved useful in experimental characterization of partially coherent light beams [37].

Returning now to the theory of communication modes and making use of Eqs. (33) and (34), the effective degree of coherence in the transmitting aperture thus is

$$\mu_0^2 = \frac{\sum_{ml}|A_{ml}|^2}{(\sum_m A_{mm})^2}, \tag{38}$$

an expression entirely in terms of the expansion coefficients of the communication modes.

We can derive similar results for the spectral density and the squared absolute value of the cross-spectral density given by Eq. (24), integrated over the observation domain O. The effective degree of coherence in the receiving aperture then takes on the form

$$\mu^2 = \frac{\sum_{ml}|g_m|^2|g_l|^2|A_{ml}|^2}{(\sum_m |g_m|^2 A_{mm})^2}. \tag{39}$$

It is interesting to note that the effective degree of coherence in the observation aperture can be obtained directly from the expansion coefficients of the communication modes in the transmitting aperture and the coupling coefficients associated with the optical system.

7. NUMERICAL EXAMPLES

As an illustration, we use the communication modes for modeling the diffraction of a partially coherent Gaussian Schell-model beam in free space between two small apertures in a near-field geometry. For simplicity, we restrict ourselves to a y-invariant situation (which could represent s-polarized light, i.e., the electric field pointing in the y direction [12]). In this case the source field is defined as

$$W(x_1,x_2) = \exp\left(-\frac{x_1^2+x_2^2}{w_0^2}\right)\exp\left[-\frac{(x_1-x_2)^2}{2\sigma_0^2}\right], \tag{40}$$

where w_0 is the half-width and σ_0 is the coherence width of the beam. The propagation of the coherent fields in the

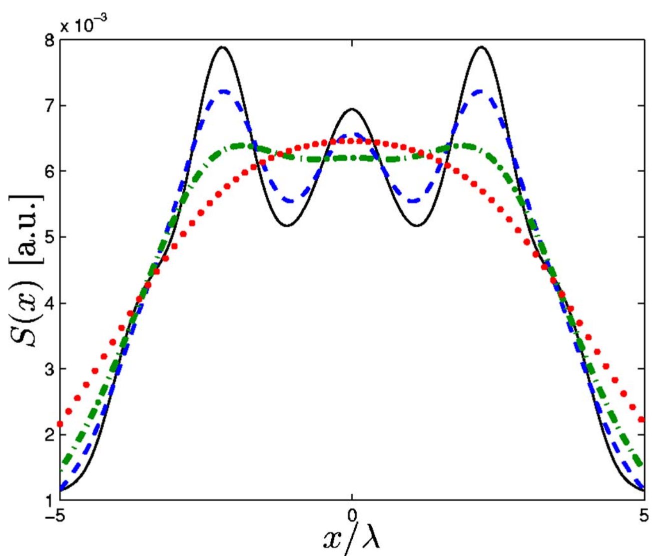

Fig. 1. (Color online) Intensity distributions of Gaussian Schell-model beams of width $w_0=10\lambda$ and coherence $\sigma_0=\infty$ (solid curve, black), $\sigma_0=5\lambda$ [dashed curve (blue online)], $\sigma_0=2\lambda$ [dash–dotted curve (green online)], and $\sigma_0=\lambda$ [dotted curve (red online)] in the receiving aperture.

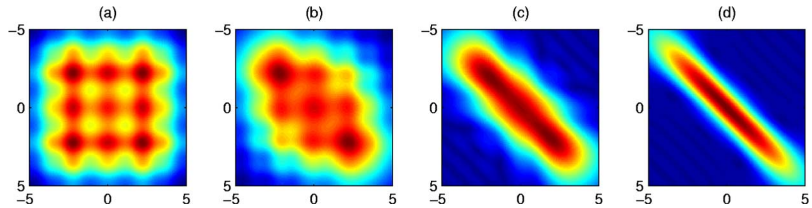

Fig. 2. (Color online) Absolute values of cross-spectral densities of Gaussian Schell-model beams with $w_0=10\lambda$, and (a) $\sigma_0=\infty$, (b) $\sigma_0=5\lambda$, (c) $\sigma_0=2\lambda$, and (d) $\sigma_0=\lambda$ in the receiving aperture.

system is accurately governed by Eq. (1) with the one-dimensional diffraction kernel [38,39]

$$G(x,\rho_x) = \frac{\mathrm{i}kz}{2}\frac{H_1^{(1)}\{k[(x-\rho_x)^2+z^2]^{1/2}\}}{[(x-\rho_x)^2+z^2]^{1/2}}, \tag{41}$$

where $k=\omega/c=2\pi/\lambda$ is the wave vector of the light and $H_1^{(1)}$ is the Hankel function of the first kind and order one. Based on the theory presented in Section 2, the coherent communication modes can be solved numerically as the singular functions in the biorthogonal expansion of the propagation operator. The corresponding communication modes and coupling coefficients for the partially coherent wave field are then simply obtained from Eqs. (19)–(21), and the diffraction of the light beam on exiting the transmitting aperture can be evaluated as described in Section 4.

We have chosen the transmitting and receiving apertures to have identical widths, $A=O=10\lambda$, where λ is the wavelength, and the distance between the apertures is also $z=10\lambda$. Figure 1 illustrates the normalized intensity distributions of various Gaussian Schell-model beams with different coherence widths and $w_0=10\lambda$ in the receiving aperture. The results show a clear dependence between the diffraction characteristics and the coherence of the field. The absolute values of the cross-spectral densities of the same fields, calculated by means of the communication modes, are illustrated in Fig. 2. Though not shown, we have checked numerically that the same results are obtained if the field is represented using the coherent modes and these are propagated individually through the system.

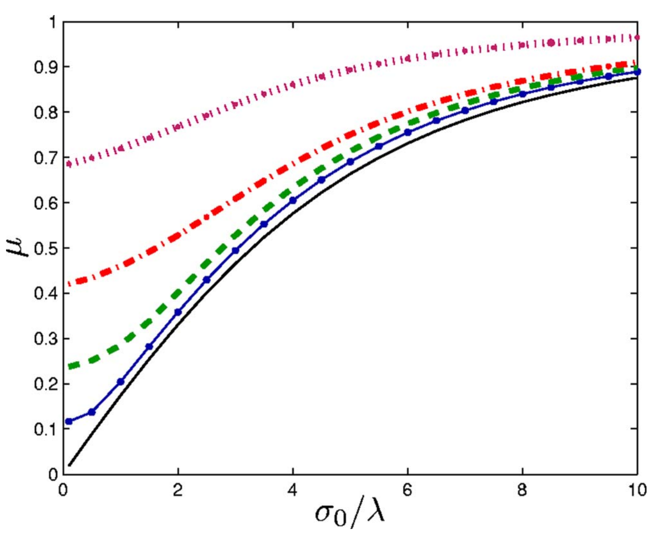

Fig. 3. (Color online) Effective degrees of coherence in the transmitting aperture (solid curve, black) and in the receiving aperture as a function of the coherence width σ_0 of Gaussian Schell-model beams with $w_0=10\lambda$. The observation-domain curves correspond to different propagation distances between the apertures: $z=10\lambda$ [solid curve with dots (blue online)], $z=25\lambda$ [dashed curve (green online)], $z=50\lambda$ [dashed–dotted curve (red online)], and $z=100\lambda$ [dotted curve (magenta online)].

We also demonstrate the use of the communication modes for determining the effective degree of coherence of Gaussian Schell-model beams in the receiving apertures of similar systems as described above. In Fig. 3, the results obtained from Eq. (39) are illustrated as a function of the coherence width of the beam for different propagation distances. The communication modes must be solved separately for the systems with different propagation distances, but the changes in the beam coherence are simply governed through the expansion coefficients. The size of the apertures is assumed to remain the same in all cases. Thus, as the beam is spreading on propagation, its effective degree of coherence in the receiving aperture gets higher with increasing propagation distances. For comparison, we also show the effective degree of coherence in the transmitting aperture.

8. CONCLUSIONS

We have developed a general representation of partially coherent wave fields using the appropriate communication modes. For deterministic optical systems, these modes are completely coherent and can be simply expressed in terms of the coherent communication modes of the same optical system. The relationship between the communication modes and the conventional coherent modes of partially coherent wave fields is elucidated. We have also shown that the effective degree of coherence can be expressed in terms the mode projection coefficients of the source field and the mode coupling strengths associated with the system, providing a convenient means of assessing the effect of the optical system on the overall coherence of the field. The results are illustrated by near-field GSM beam calculations, demonstrating the accuracy and versatility of the communication modes method for assessing partially coherent wave fields. Further work could include developing the theory for random media and various fluctuating optical systems.

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